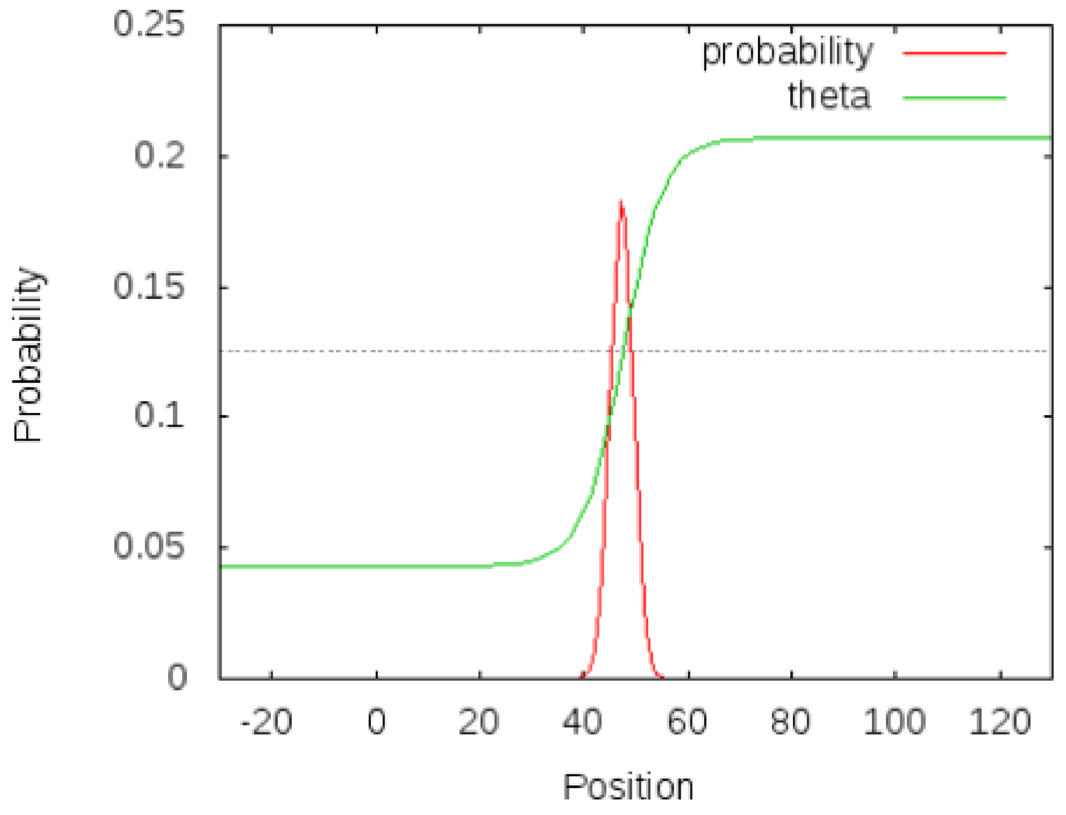
トポロジカル相と対称性を活用した量子状態制御
--- 量子ウォークによるアプローチ ---

小布施　秀明 (北海道大学)



HOKKAIDO UNIVERSITY

Topological Materials Science

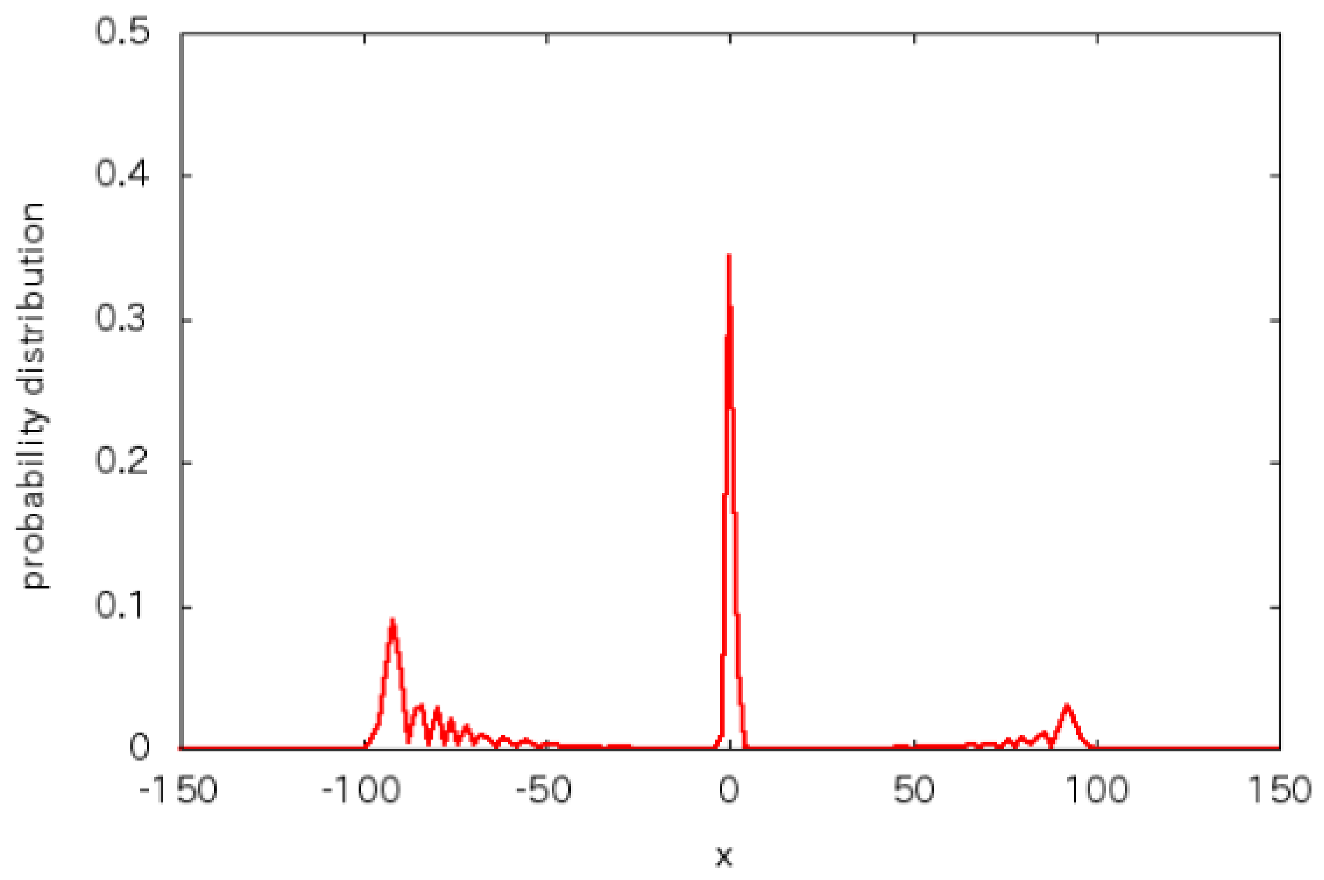
量子情報・物性の新潮流
8月3日, 2018 @ 東京大学物性研究所

はじめに

My background : 物性理論
(トポロジカル絶縁体、アンダーソン局在)

量子ウォーク:量子情報・量子計算の分野で発展, 量子シミュレーター

量子ウォークを用いて、トポロジカル相に起因するエッジ状態を誘起



- 系の対称性を制御
- 非自明なトポロジカル相が容易に誘起
- 検証実験が可能
- エッジ状態の実空間観測
- (現在の)電子系の実験では困難なセットアップも可

トポロジカル相と対称性を活用した量子状態制御

Outline

- 量子ウォークとは
- 量子ウォークにおけるトポロジカル相とエッジ状態
- パリティ-時間対称な開放系量子ウォークにおけるトポロジカル相
（非エルミート系におけるトポロジカル相）
- 動的変化するトポロジカル相を用いた量子状態輸送

Acknowledgements

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河崎　真樹男 (M1)
金　多景 (学部卒→ ETH)
清水　洋祐 (修士卒)

- 実験:

Peng Xueグループ (東南大学、中国)

unitary QW:

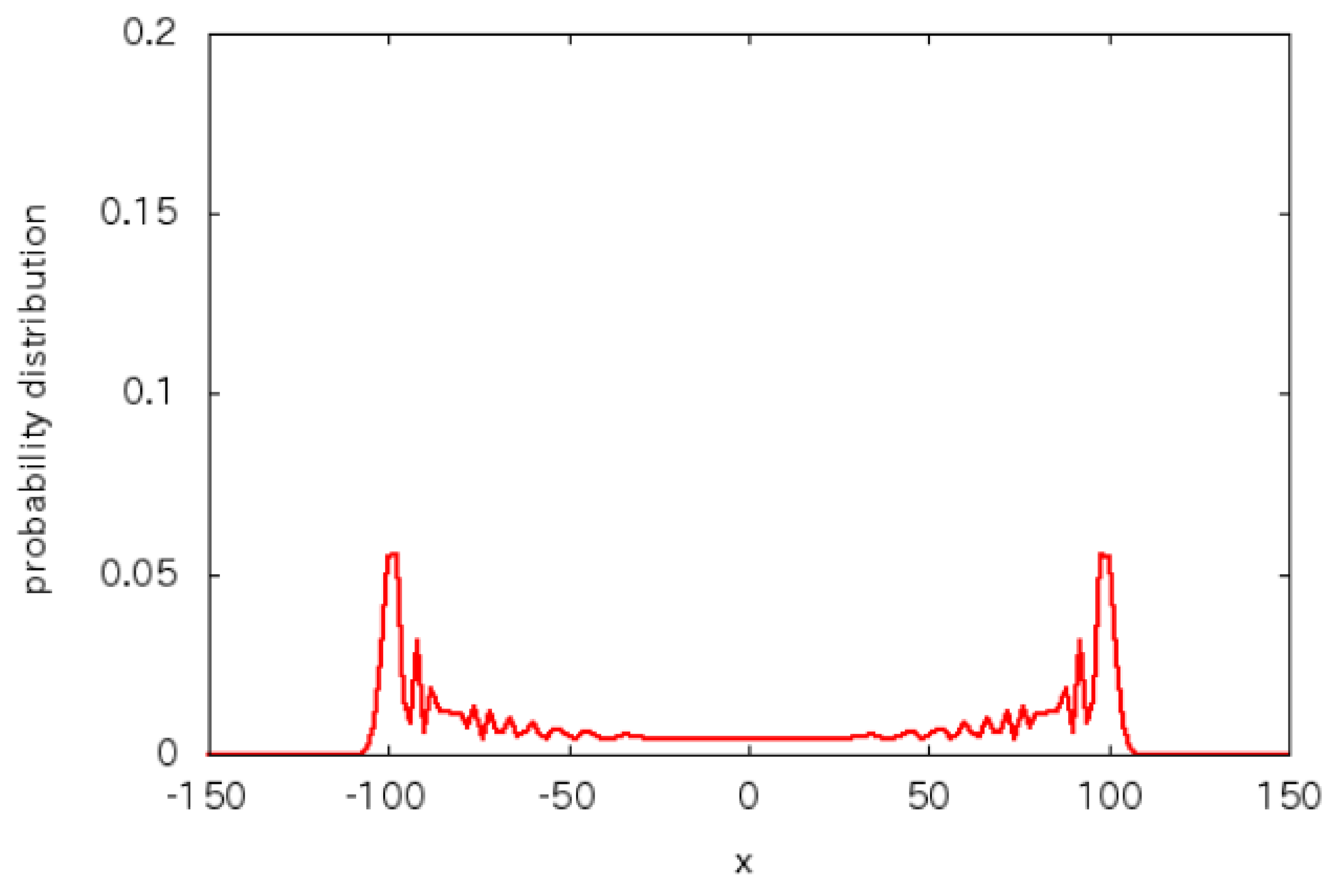
PRB 84, 195139 (2011).
PRB 88, 121406(R) (2013).
PRB 92, 045424 (2015).

$\mathcal{PT}$ symmetric non-unitary QW:

PRA 93, 062116 (2016).
IIS 23, 95 (2017) [arXiv:1608.00719]
arXiv:1609.09650.
Nature Physics 13, 1117 (2017)

量子ウォーク(Quantum walk, QW)とは？

- QW has been developed in the field of
  quantum computation & information in 90's.
  universal quantum computer & Grover algorithm
- Synthesis quantum system; realized in
  photonic/optical systems, cold atoms, etc.
- In *a classical limit*, the dynamics of QWs
  is identical with that of random walks.
- QWs are also useful as quantum simulators.

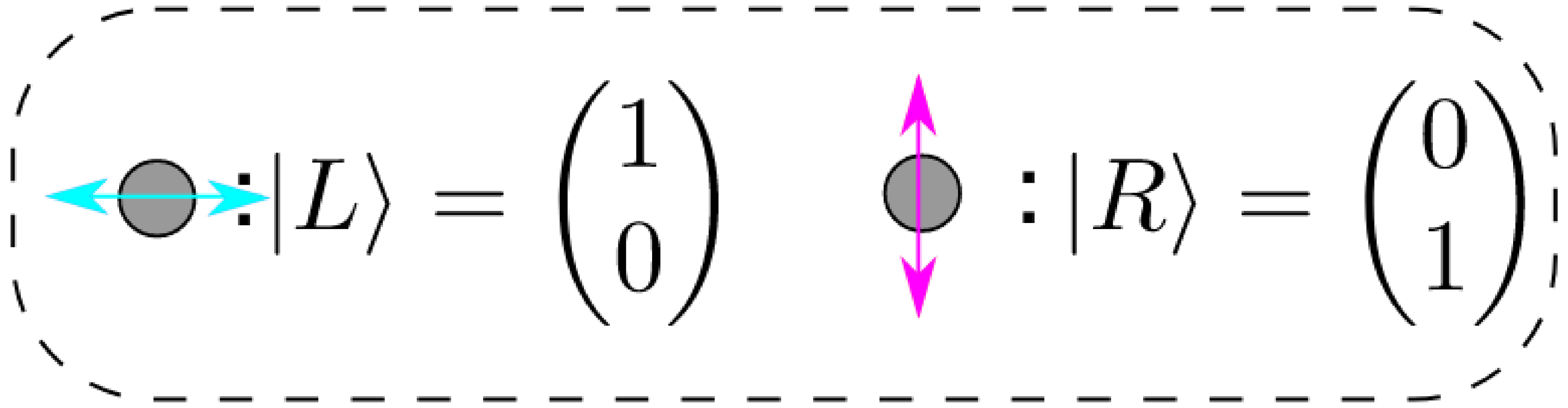


Definition of Discrete-Time QW in 1D

1.Basis:

- position space $\otimes$ internal states

$$|x\rangle \otimes |s\rangle \qquad x \in \mathbb{Z}, \quad s = L, R$$

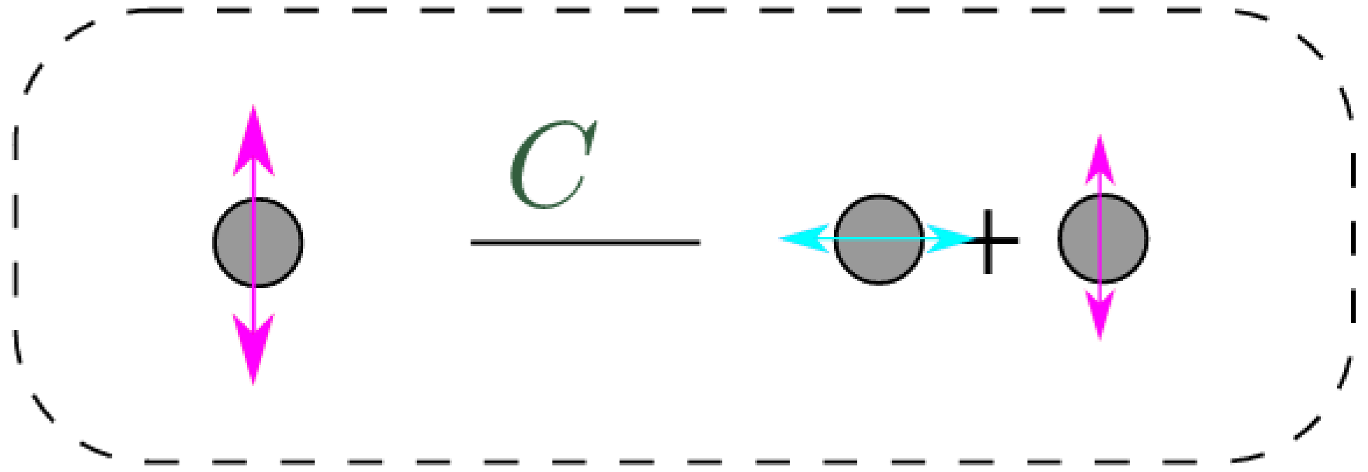


2. Defining elemental unitary operators:

- Coin operator: C

$$C[\theta(x)] = \sum_x |x\rangle\langle x| \otimes \mathcal{R}[\theta(x)]$$

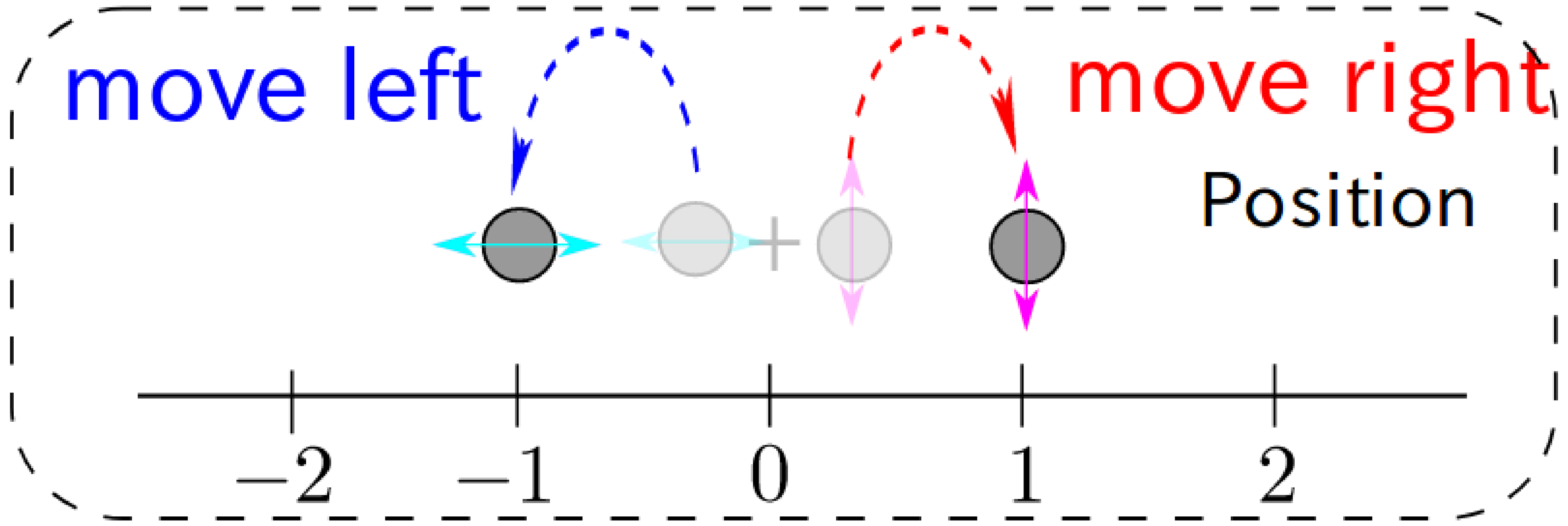
$$\mathcal{R}(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{pmatrix} = e^{-i\theta\sigma_2}$$



- Shift operator : S

$$S = \sum_x \Big(|x+1\rangle\langle x| \otimes |R\rangle\langle R| + |x-1\rangle\langle x| \otimes |L\rangle\langle L| \Big)$$

$$\stackrel{\mathcal{F}}{\rightarrow} \int dk |k\rangle\langle k| \otimes e^{ik\sigma_3}$$



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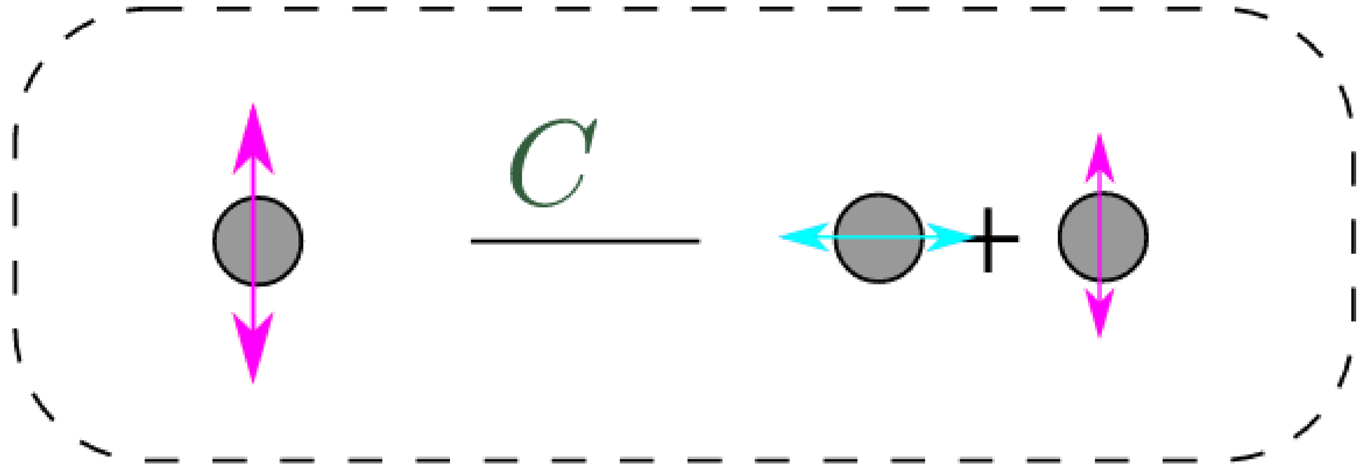

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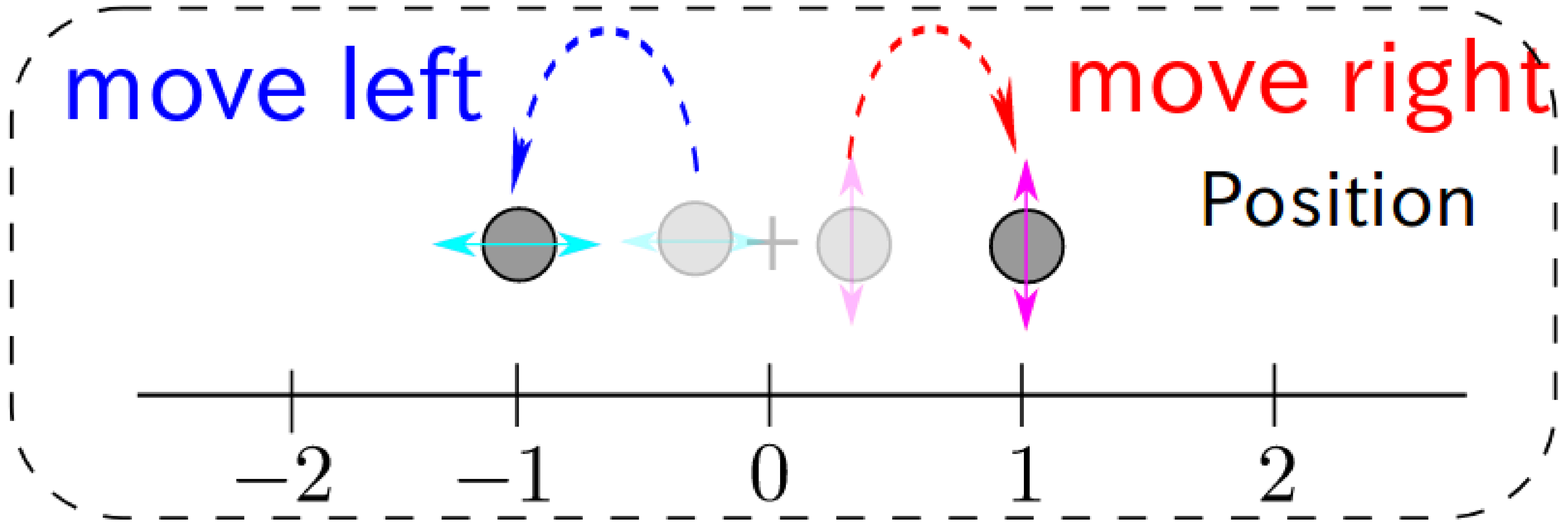
$$\xrightarrow{\mathcal{F}} \int dk |k\rangle\langle k| \otimes e^{ik\sigma_3}$$



3. Building up time-evolution operators by products of elemental operators:

- Time-evo. operator for single-time step :

single-step: $U = SC(\theta)$

two-step: $U = SC(\theta_2) \cdot SC(\theta_1)$

- quantum state at time t $(\in \mathbb{Z}^+)$

$$|\psi(t)\rangle = U^t |\psi(0)\rangle$$

periodically and stroboscopically driven Floquet systems

3. Building up time-evolution operators by products of elemental operators:

- Time-evo. operator for single-time step :

single-step: $U = SC(\theta)$

two-step: $U = SC(\theta_2) \cdot SC(\theta_1)$

Quantum dynamics can be defined by U.

➡ No Hamiltonian is explicitly defined.

$$U = e^{-iH_F}$$

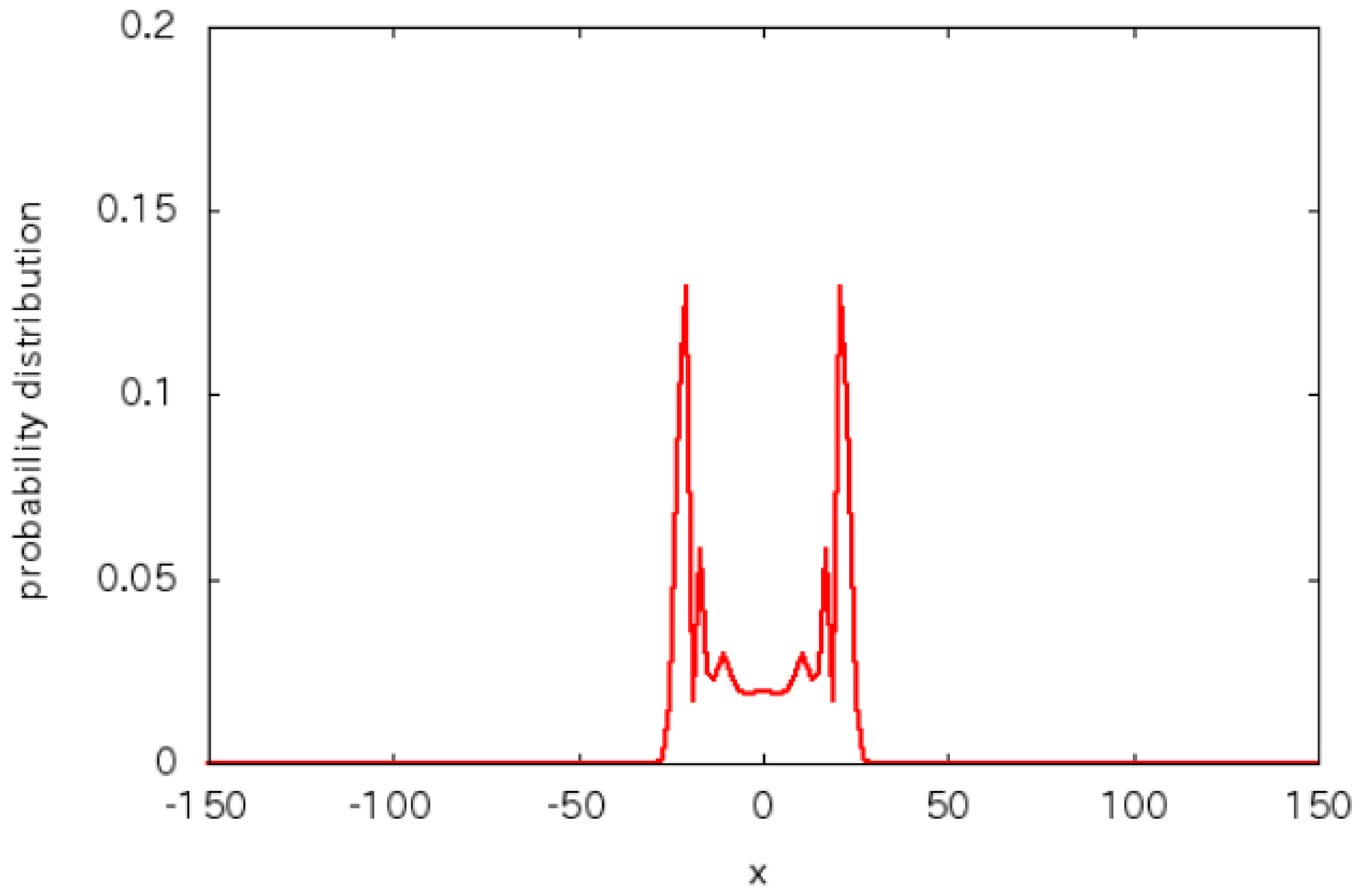
- Continuum limit

$U = SC(\theta) \longrightarrow$ Dirac operator $H_F = \hat{p}\sigma_3 + \theta(x)\sigma_2$

- quantum state at time $t\ (\in \mathbb{Z}^+)$

$$|\psi(t)\rangle = U^t |\psi(0)\rangle$$

periodically and stroboscopically driven Floquet systems



Quasi-energy

- Stationary states (eigen states of U):

$$U|\psi\rangle = \lambda|\psi\rangle \qquad U = e^{-iH_F}$$

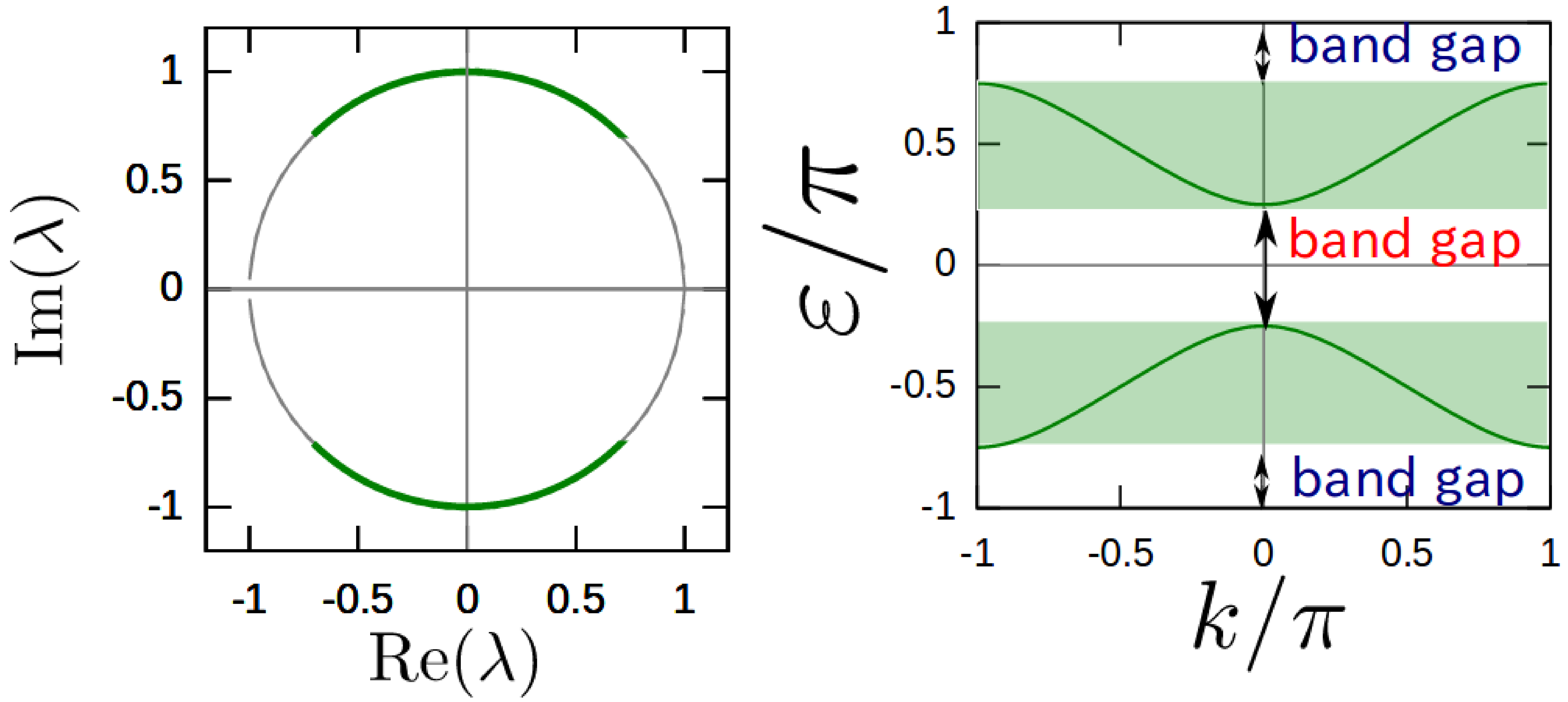
$$\lambda = e^{-i\varepsilon}$$

ε : quasi-energy

(2π periodicity)

- For unitary QWs,

$$|\lambda| = 1 \Rightarrow \varepsilon \in \mathbb{R}$$

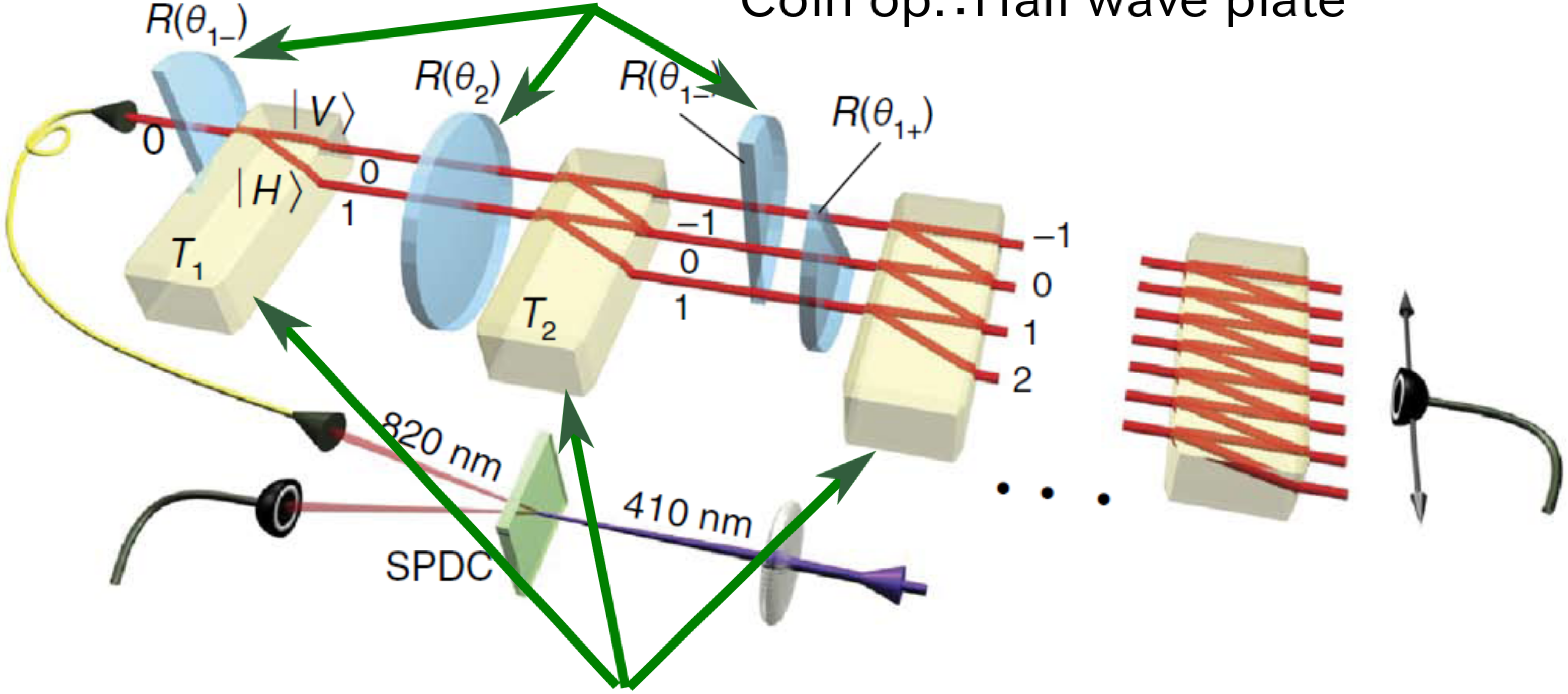


Experiment: bulk optics with photons

internal states :
vertical & horizontal polarizations

Kitagawa, Broome, Fedrizzi *et al.*, Nature Comm. **3**, 882 ('12)

Coin op.:Half wave plate

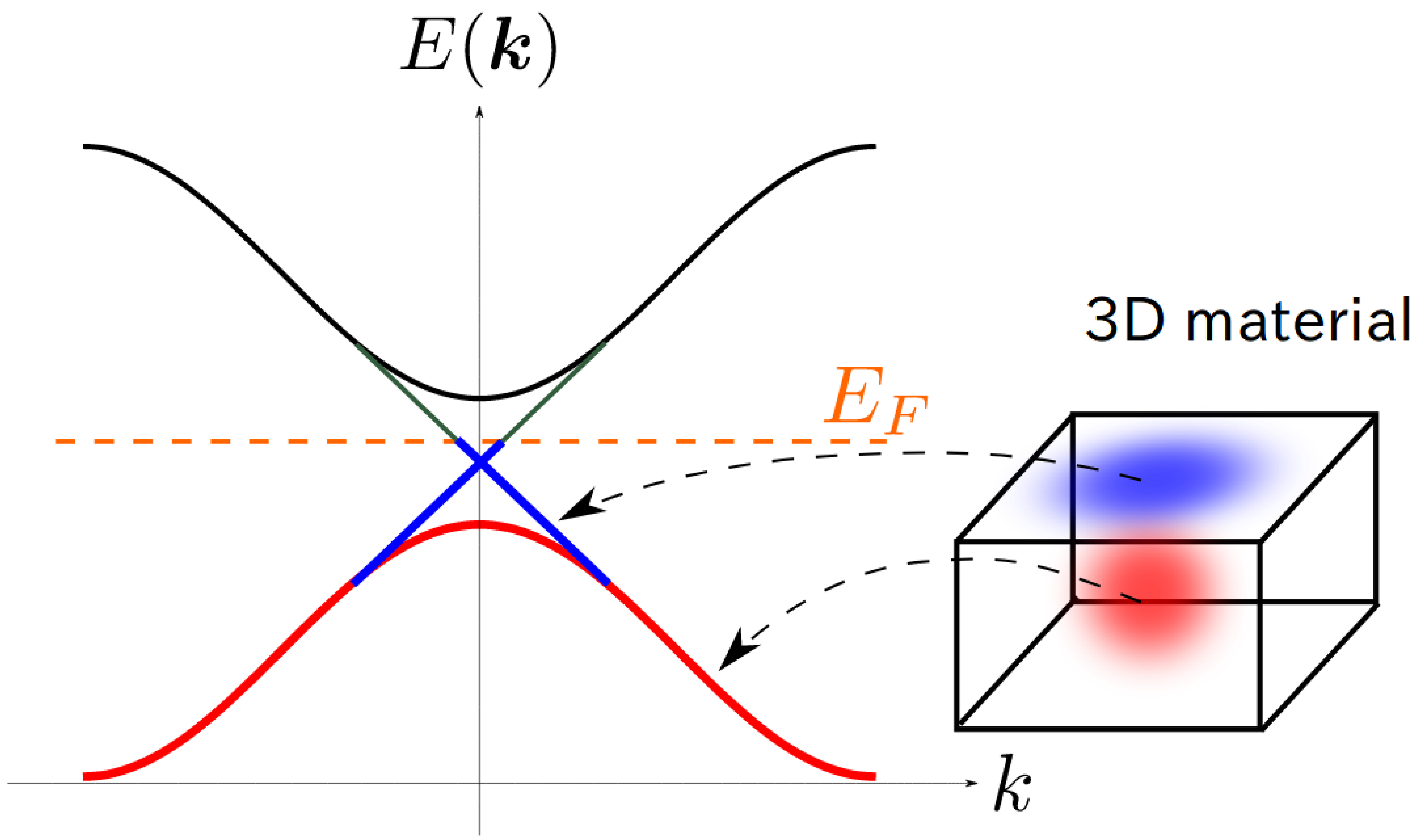


Shift op.:Beam displacer

- 量子ウォークにおけるトポロジカル相とエッジ状態

Topological insulator

theoretical prediction：Kane, Mele, PRL (05)



Bulk states： → Insulating

E_F in the band gap

Surface (edge) states： → Metallic

spectrum in the bulk band gap (dissipationless)

E_F in the continuous spectrum

Symmetry for topological phases

- Time-reversal symm.

$$\mathcal{T}H\mathcal{T}^{-1} = H$$

$\mathcal{T}$: anti-unitary

$$\mathcal{T}^2 = \pm 1$$

- Particle-hole symm.

$$\Xi H \Xi^{-1} = -H$$

Ξ : anti-unitary

$$\Xi^2 = \pm 1$$

- Chiral symm.

$$\Gamma H \Gamma^{-1} = -H$$

Γ : unitary

Classification table:

universality class		TRS	PHS	chiral symmetry	$d=1$	$d=2$	$d=3$
Standard	A	0	0	0	-	$\mathbb{Z}$	-
(Wigner-Dyson)	AI	+1	0	0	-	-	-
	AII	−1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
	AIII	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI	+1	+1	1	$\mathbb{Z}$	-	-
	CII	−1	−1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	+1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	−1	0	-	$\mathbb{Z}$	-
	DIII	−1	+1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	+1	−1	1	-	-	$\mathbb{Z}$

Schnyder, Ryu, Furusaki, Ludwig, PRB '08; Kitaev, AIP Conf. '09

Topological Phases in QWs

Kitagawa, Rudner, Berg, and Demler, PRA ('10)
HO & Kawakami, PRB ('12).
Asbóth & HO, PRB ('13).
HO, Asboth, Nishimura, and Kawakami, PRB ('15).

- Two-step QW (unitary):

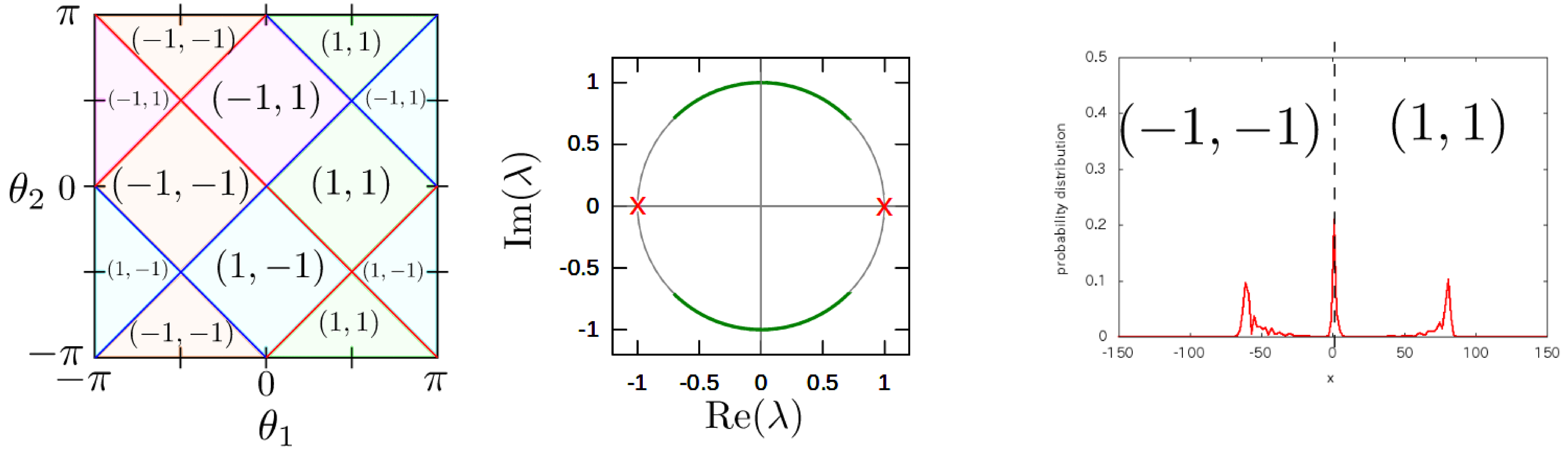
$$U = SC(\theta_2) \cdot SC(\theta_1)$$

- Symmetry class : class BDI

$$U' = C(\theta_1/2)SC(\theta_2)SC(\theta_1/2)$$ by "symmetry time frame"

- Topological numbers (ν_0, ν_π) : two edge states at $\varepsilon = 0$ & π .

winding number $$\nu' = \frac{1}{\pi i}\int_{-\pi}^{\pi} dk \langle \psi_-(k)| \frac{d}{dk} |\psi_-(k)\rangle$$



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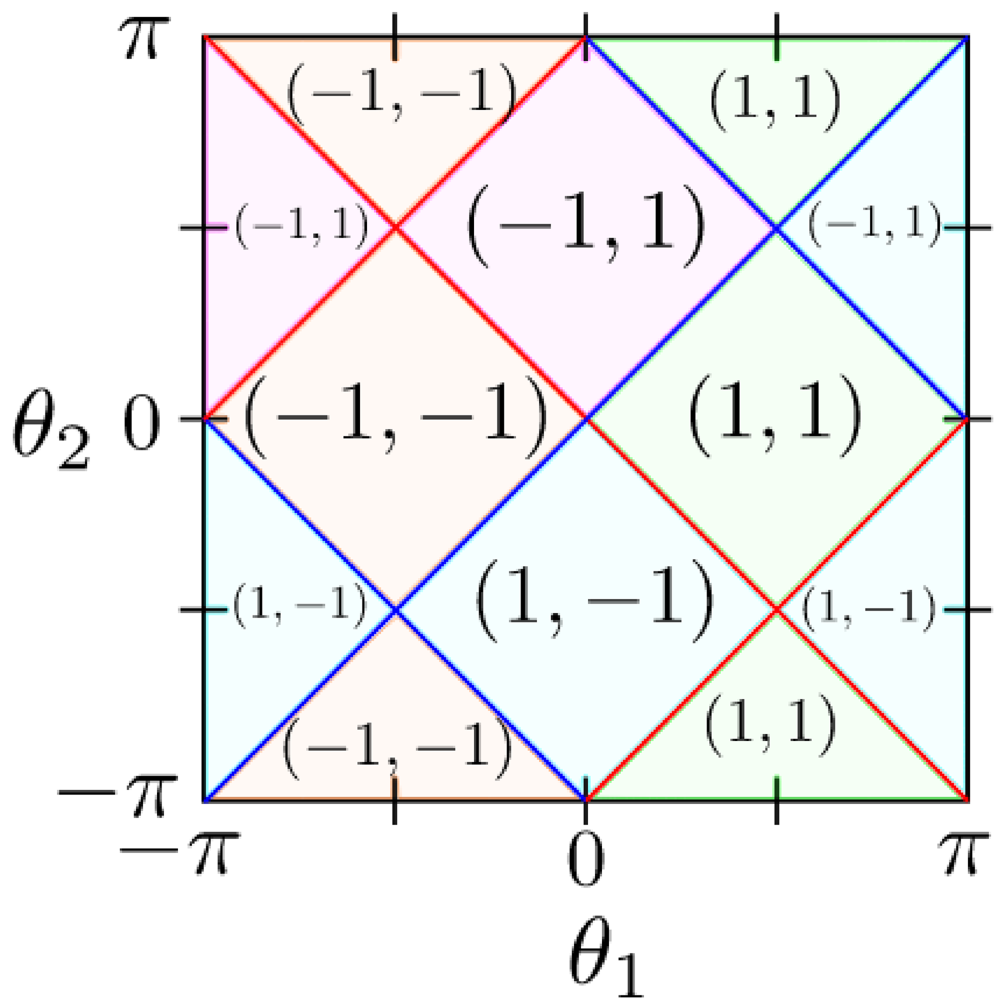
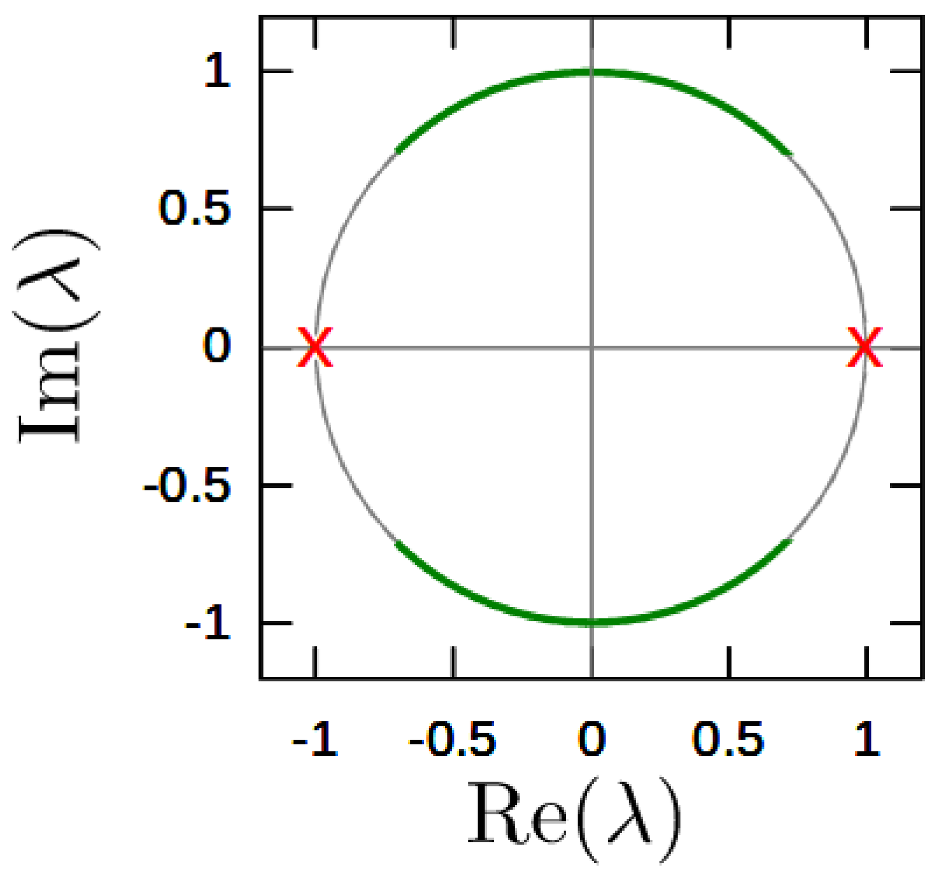
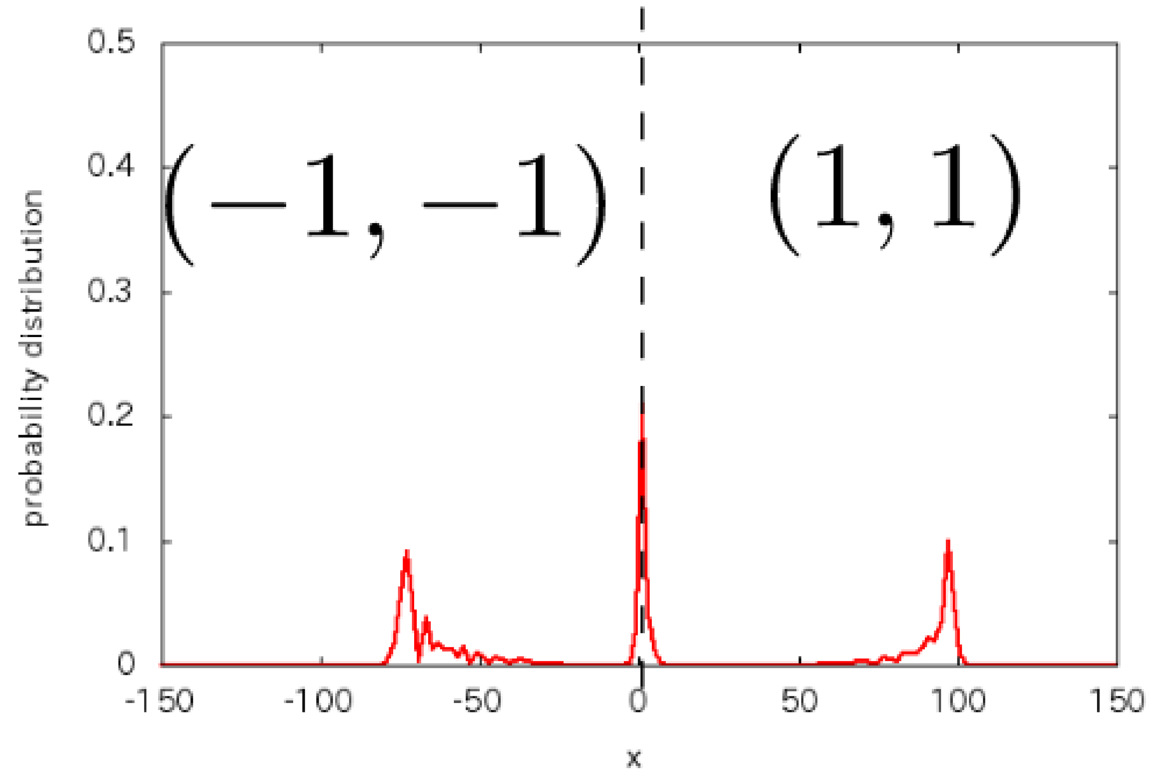
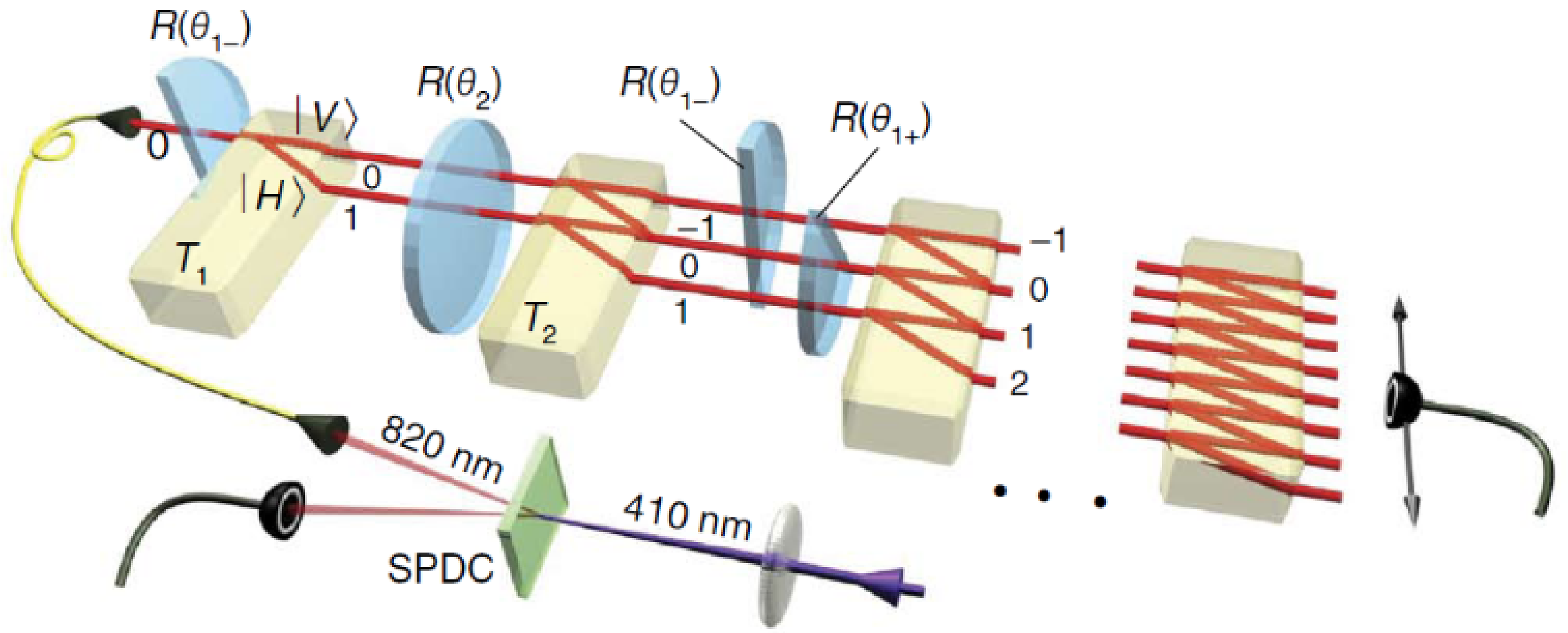


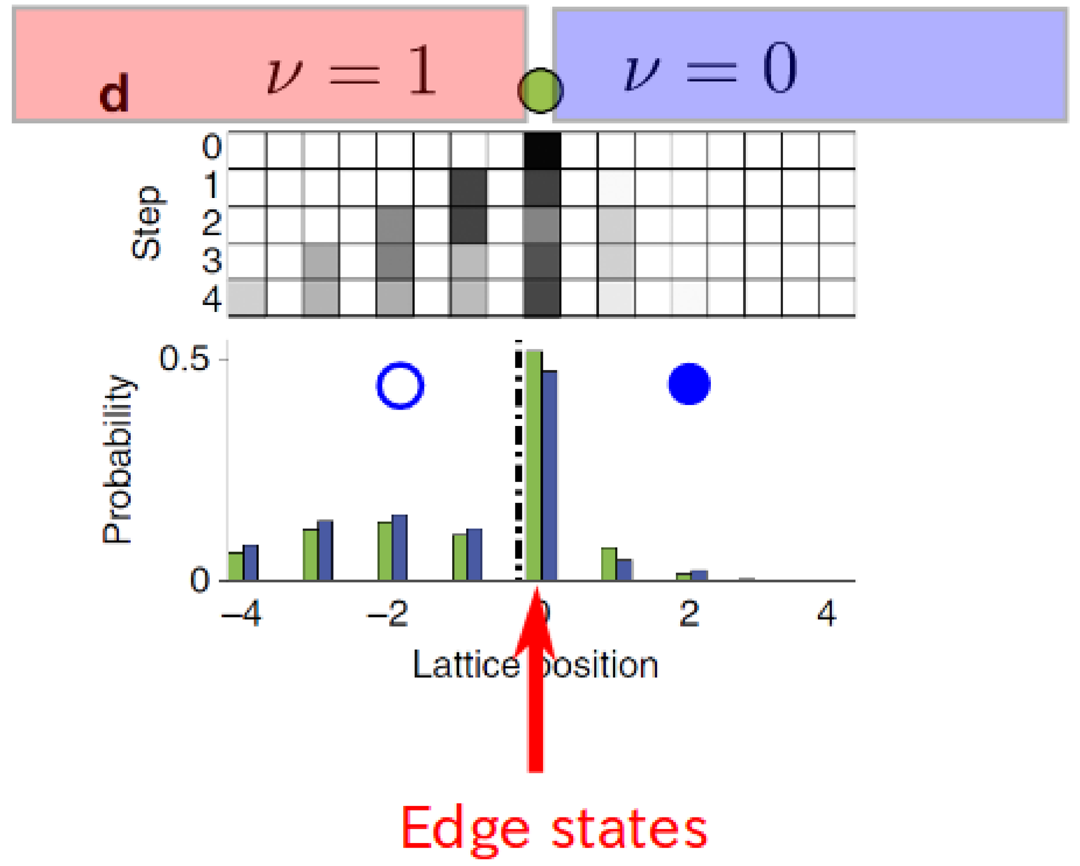




Observation of edge states in 1D QW

Kitagawa, Broome, Fedrizzi *et al.*, Nature Comm. **3**, 882 ('12)





We can predict the existence of edge states of quantum walks by using knowledge of topological insulators.

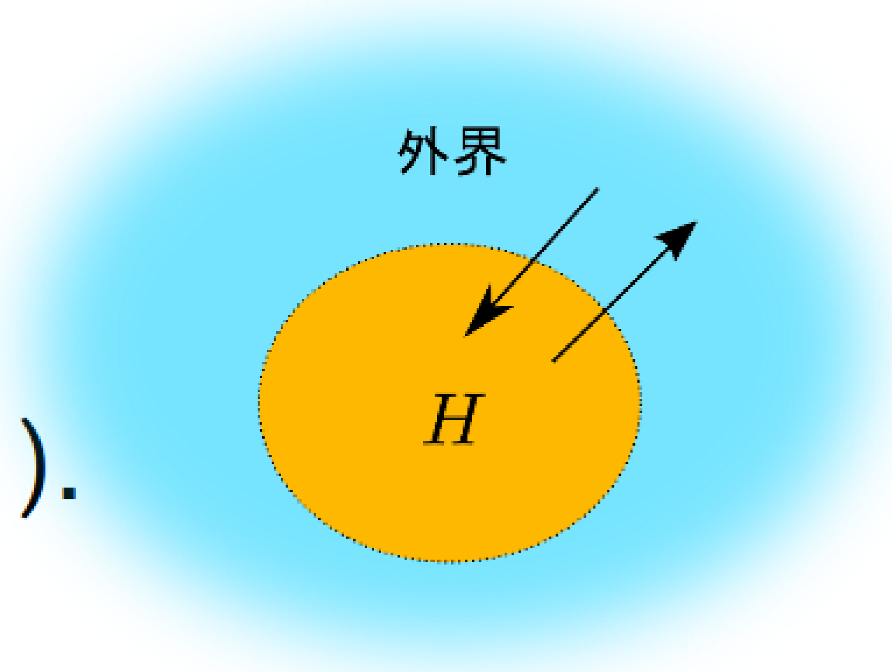
- パリティ-時間対称な開放系量子ウォークにおけるトポロジカル相
（非エルミート系におけるトポロジカル相）

光子の損失効果を用いることにより、量子ウォーク特有の現象を調べたい

$\mathcal{PT}$ Symmetry in non-Hermitian system

- Non-Hermitian system: $H \neq H^{\dagger}$

  Generally, energy becomes complex ($E \in \mathbb{C}$).



- Non-Hermitian system with $\mathcal{PT}$ symmetry [Bender and Boettcher, PRL ('98)]

  $\mathcal{P}$: Parity operator $x \to -x$

  $\mathcal{T}$: Time-reversal operator $t \to -t$

$$\mathcal{PT}\,H\,(\mathcal{PT})^{-1} = H$$
$$\mathcal{PT}|\psi\rangle = e^{i\alpha}|\psi\rangle$$

$\Rightarrow E \in \mathbb{R}$

- Interesting phenomena which never occur in closed systems.
- PT symmetric systems are realized by using classical lasers.
- However, no experiment in quantum regime.

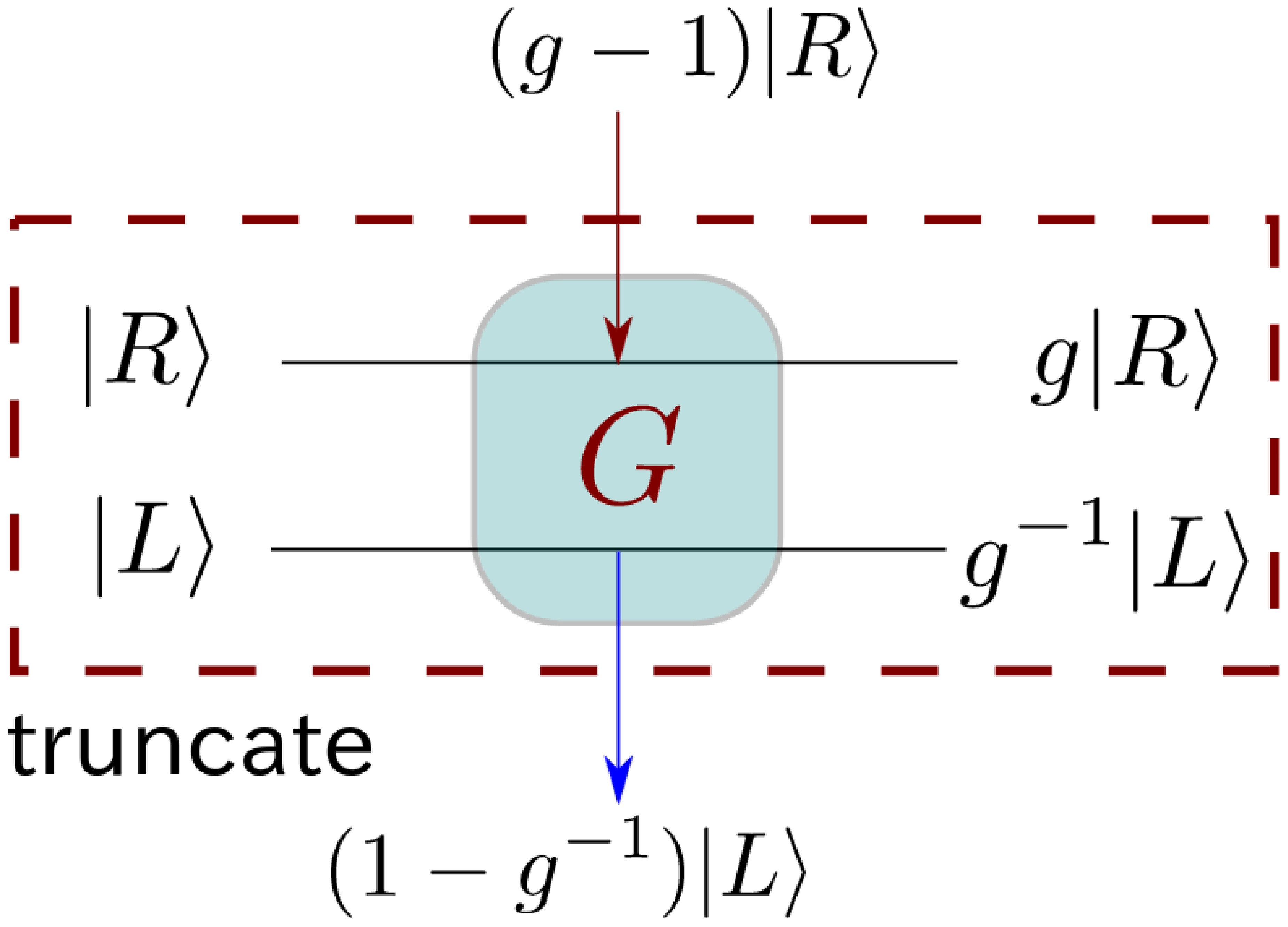
Non-unitary QWs with Gain & Loss

Mochizuki, Kim, Obuse, PRA 93, 062116 (2016).

- (phenomenological)gain & loss operators:

$$G = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} g^{-1} & 0 \\ 0 & g \end{pmatrix}$$

$(g > 1)$



- Non-unitary time-evolution operator:

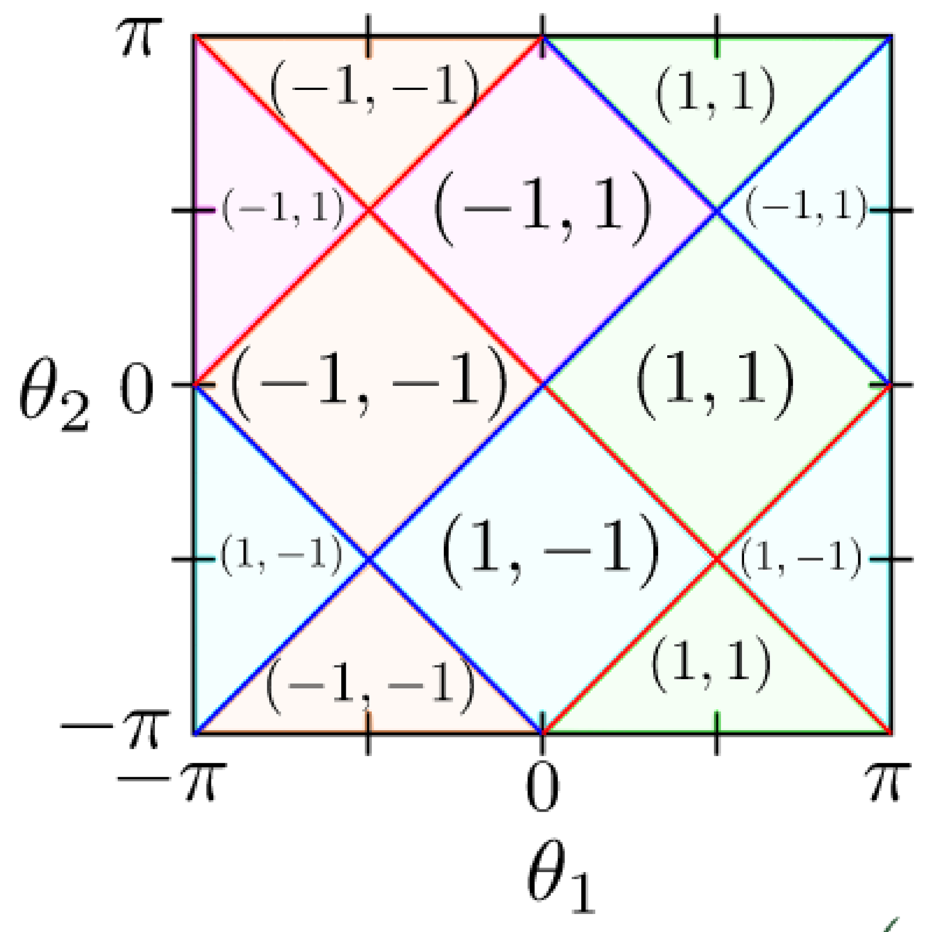
$$U_{gl} = G^{-1} S C(\theta_2) G S C(\theta_1)$$

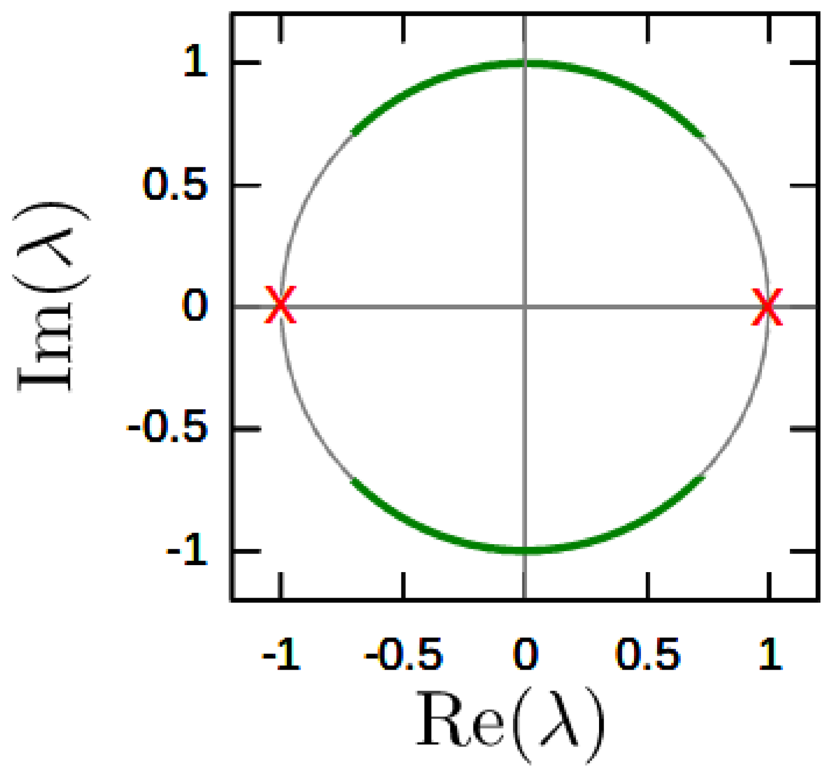
$\mathcal{PT}$ symmetry $\quad (\mathcal{PT}) U'_{gl} (\mathcal{PT})^{-1} = U'^{-1}_{gl}$

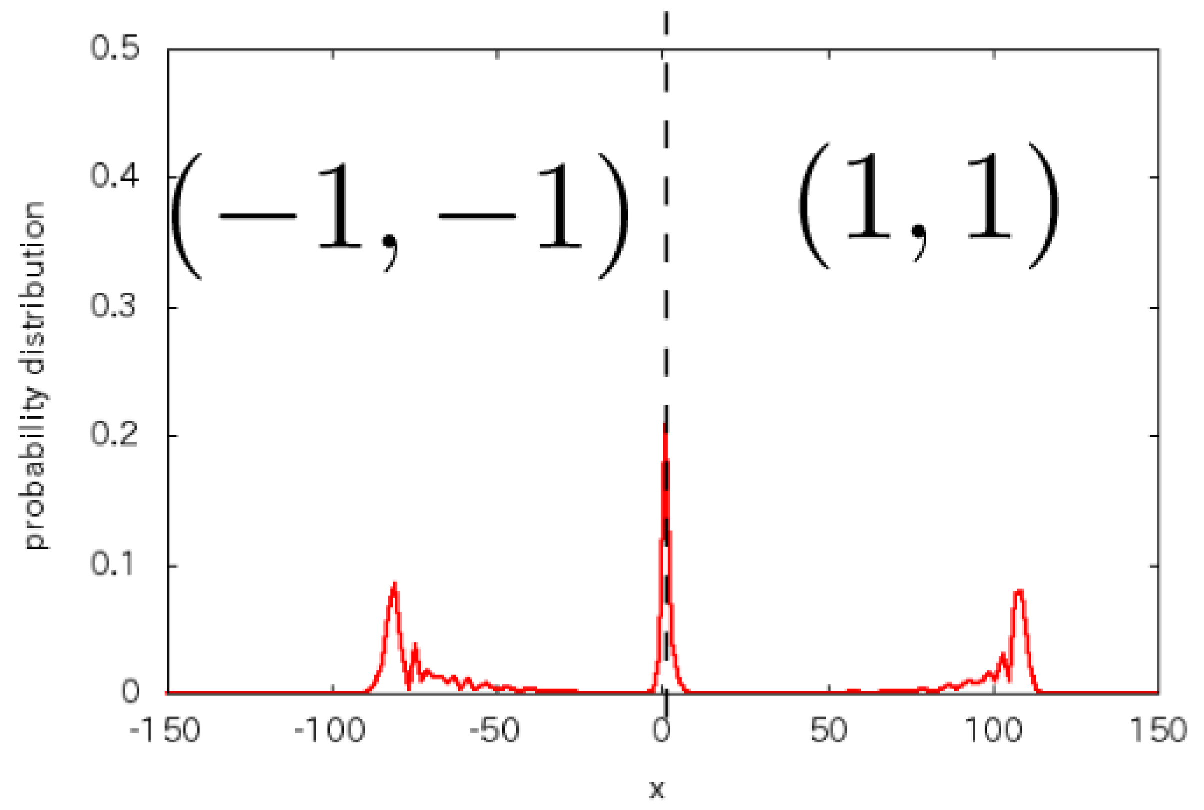
$$\mathcal{PT} = \sum |-x\rangle\langle x| \otimes \sigma_3 K$$

Floquet Topological Phases in QWs

- Unitary two-step QW $U = SC(\theta_2) \cdot SC(\theta_1)$

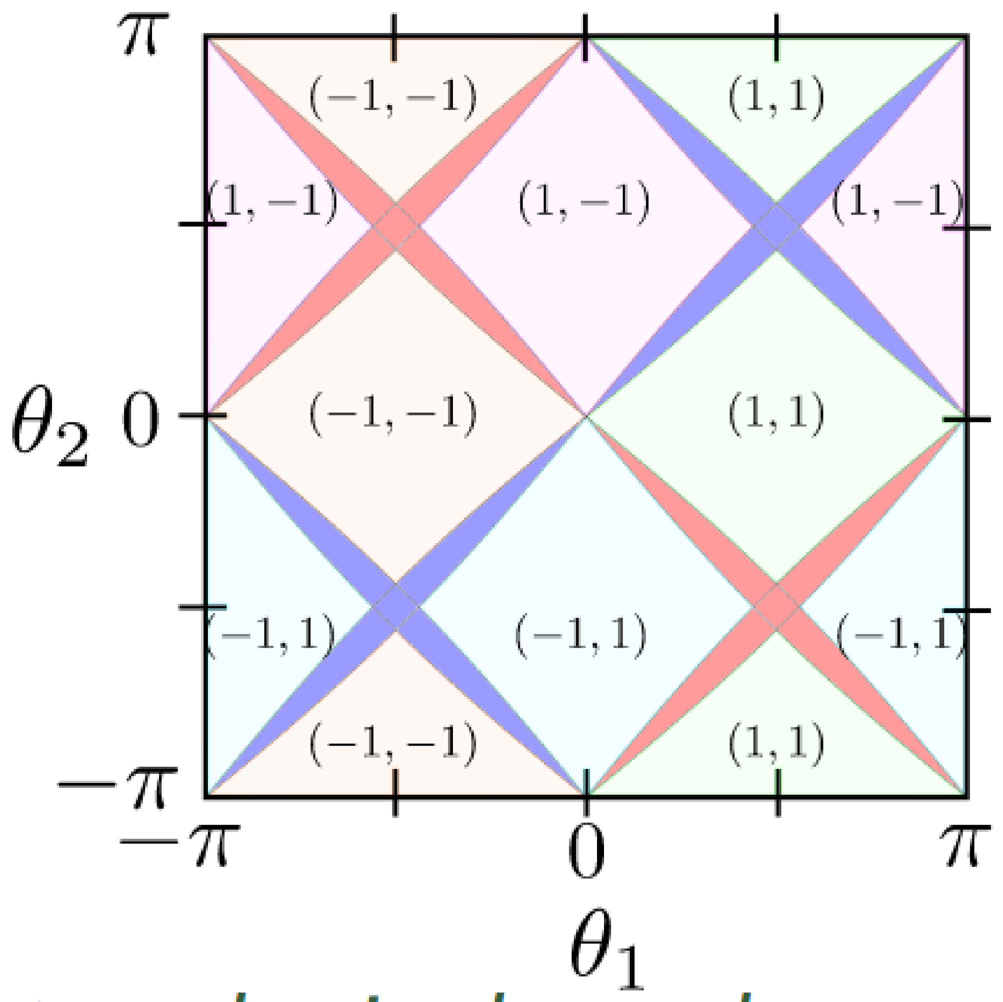




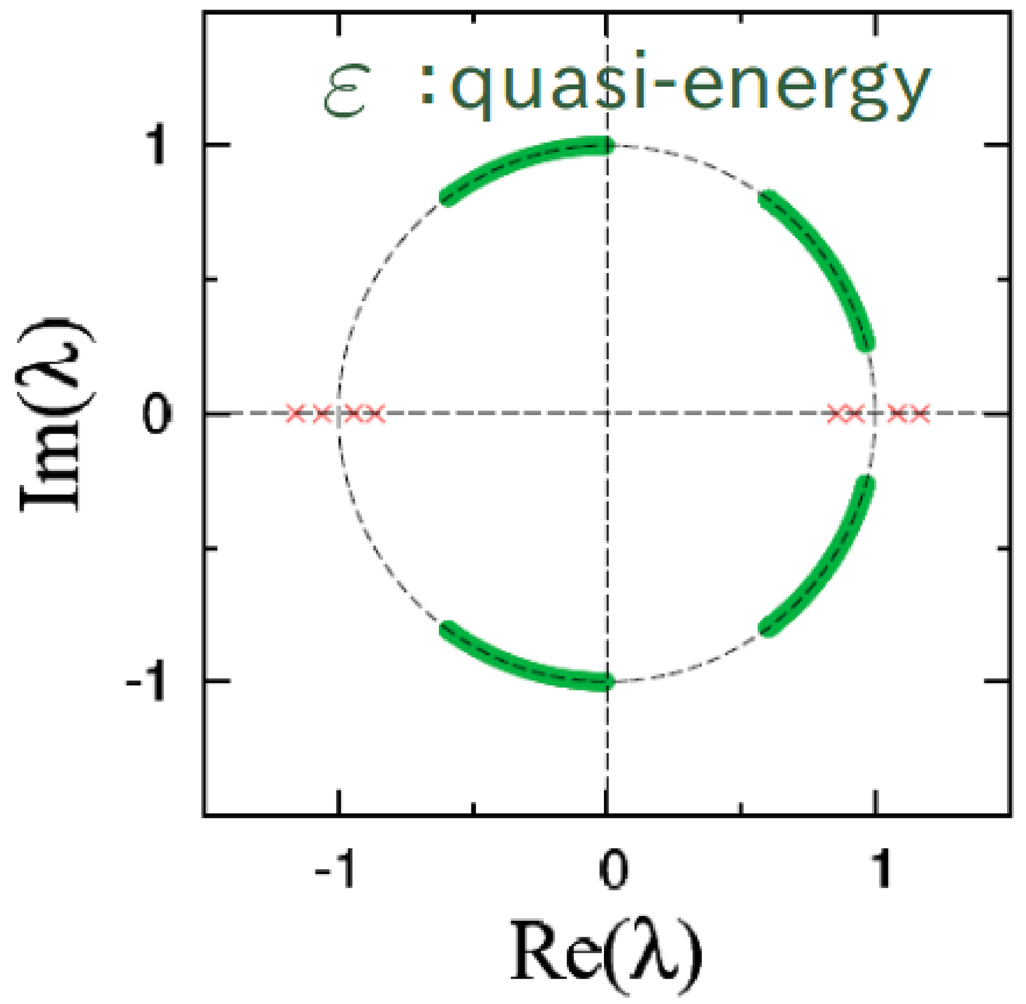


Topological numbers (ν_0, ν_π) :

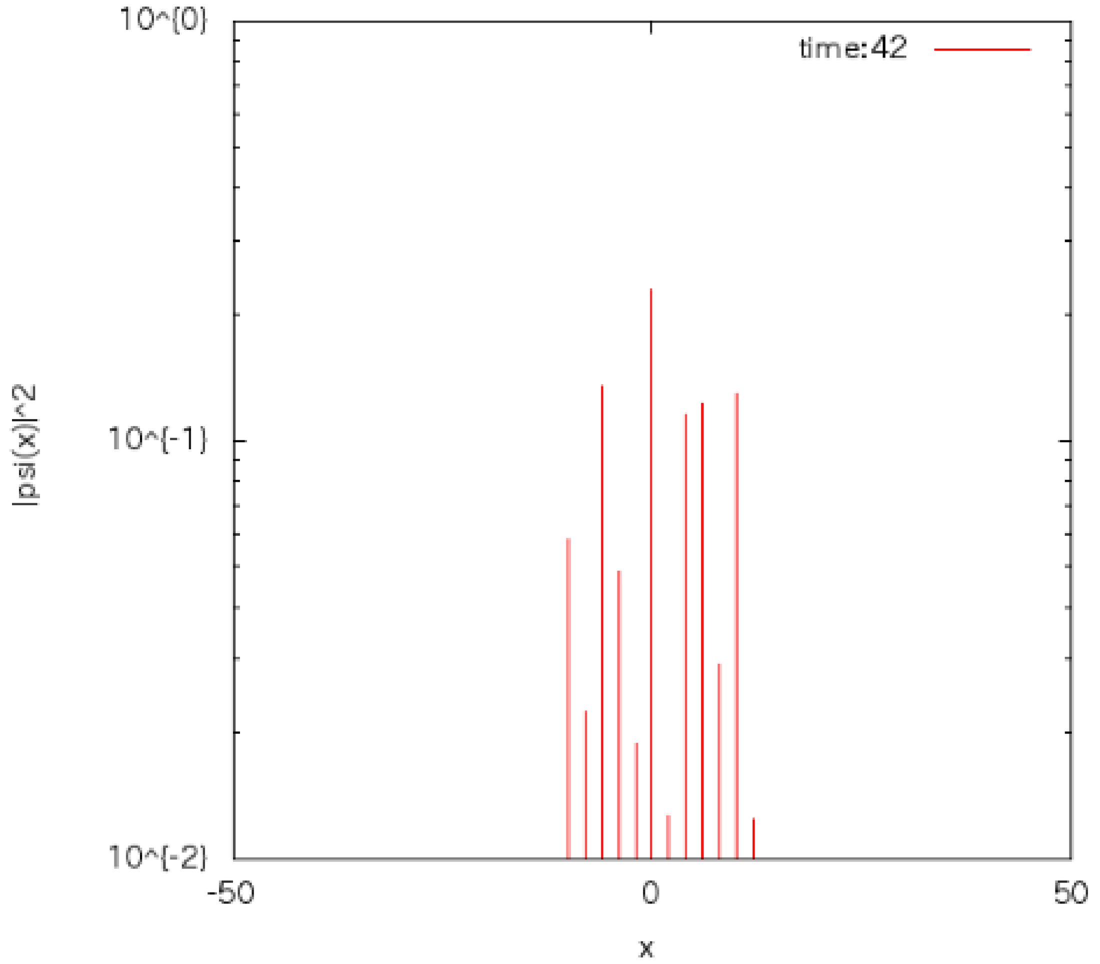
- PT symmetric non-unitary QW $U_{gl} = G^{-1}SC(\theta_2)GSC(\theta_1)$



topological numbers (ν_0, ν_π)



bulk-edge correspondence



enhancement of wavefunction amplitude

Experiment

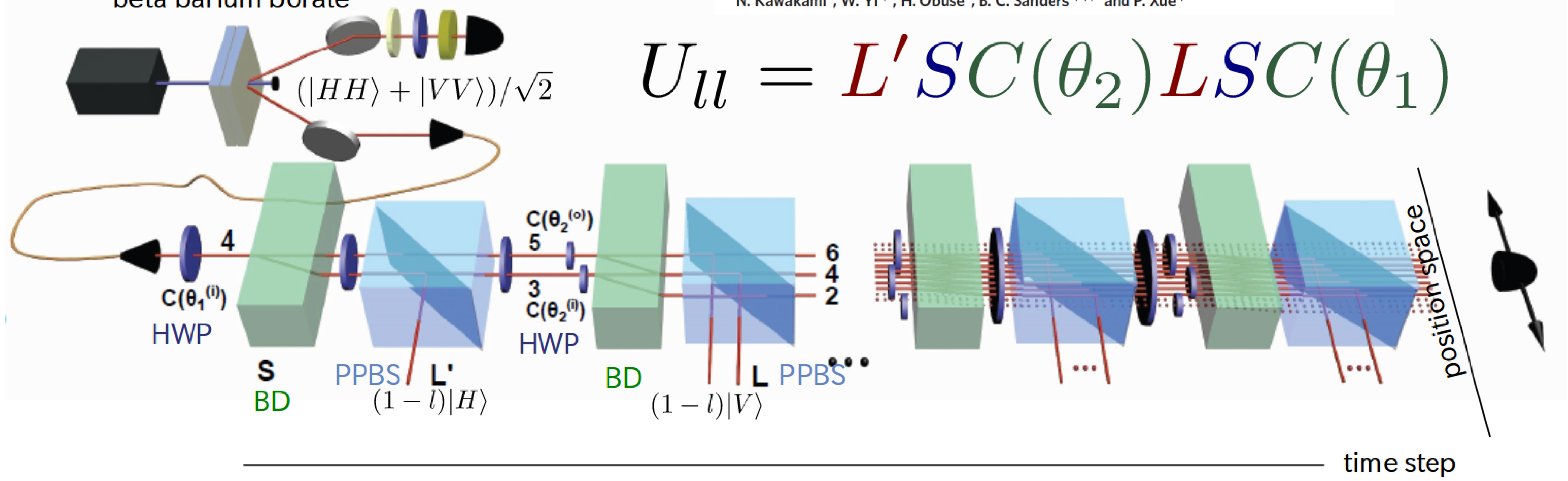
nature physics ARTICLES
PUBLISHED ONLINE: 31 JULY 2017 | DOI: 10.1038/NPHYS4204

Observation of topological edge states in parity-time-symmetric quantum walks

L. Xiao[1], X. Zhan[1], Z. H. Bian[1], K. K. Wang[1], X. Zhang[1], X. P. Wang[1], J. Li[1], K. Mochizuki[2], D. Kim[2], N. Kawakami[3], W. Yi[4,5], H. Obuse[2], B. C. Sanders[5,6,7,8] and P. Xue[1,9*]

a pair of entangled photons

$$U_{ll} = L' S C(\theta_2) L S C(\theta_1)$$



Parameters:

$$\theta_1, \theta_2, l = 0.8, 0.64$$

up to 6 time step

Delayed-choice of the initial state for entangled photon pairs

~10,000 photons

Raw probability

$$P_{\rm raw}(x,t) = \frac{\text{\#detected photons at x and t}}{\text{\# photon pairs}}$$

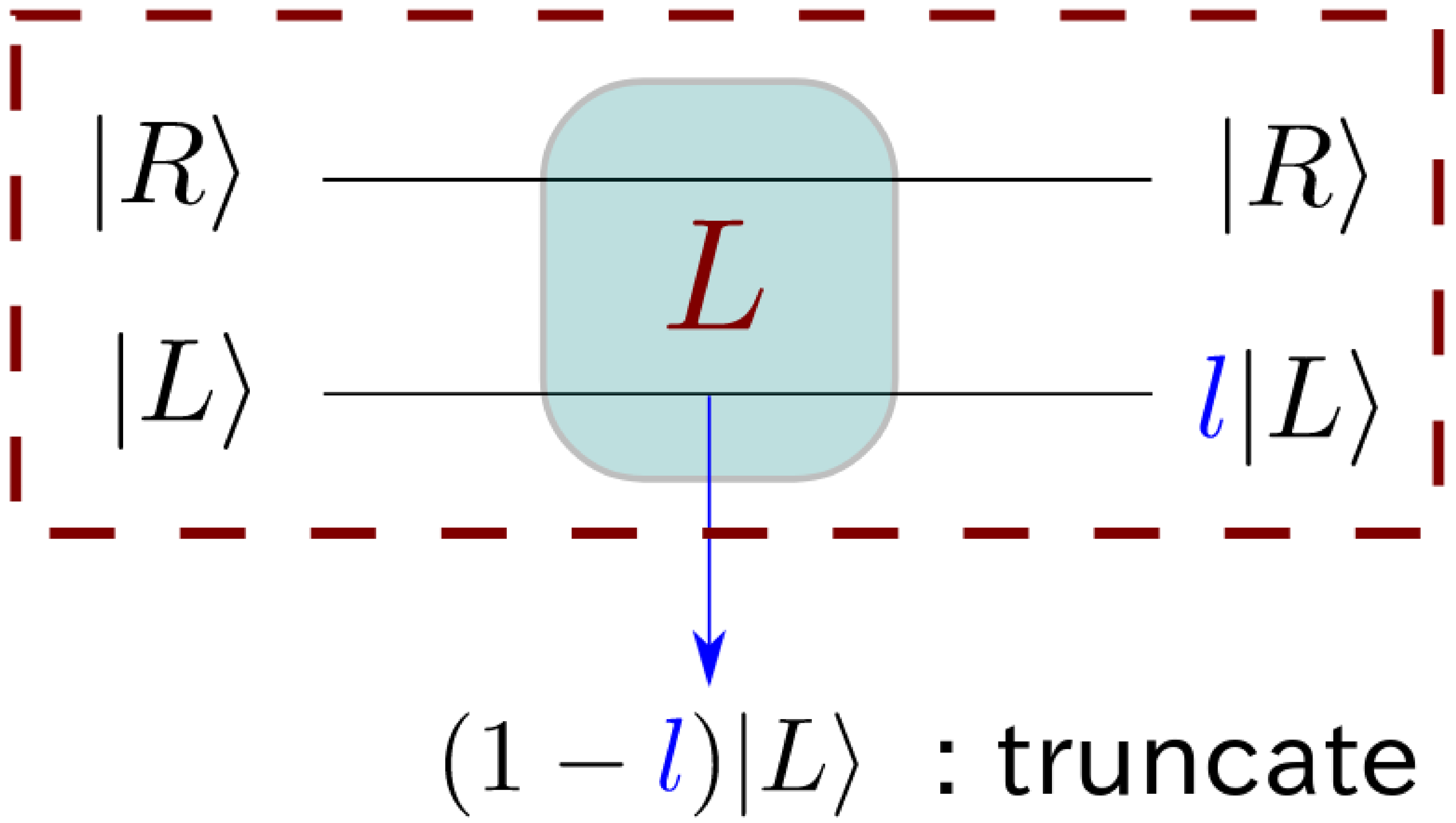
Non-unitary QWs with Loss

- Loss operators:

$$L = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} l & 0 \\ 0 & 1 \end{pmatrix}$$

$$L' = \sum_x |x\rangle\langle x| \otimes \begin{pmatrix} 1 & 0 \\ 0 & l \end{pmatrix}$$

$$(l \leq 1)$$



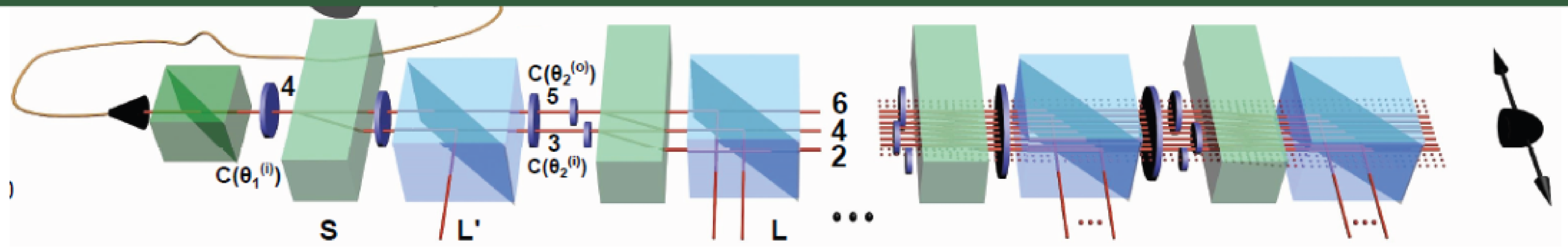
- Non-unitary time-evolution operator:

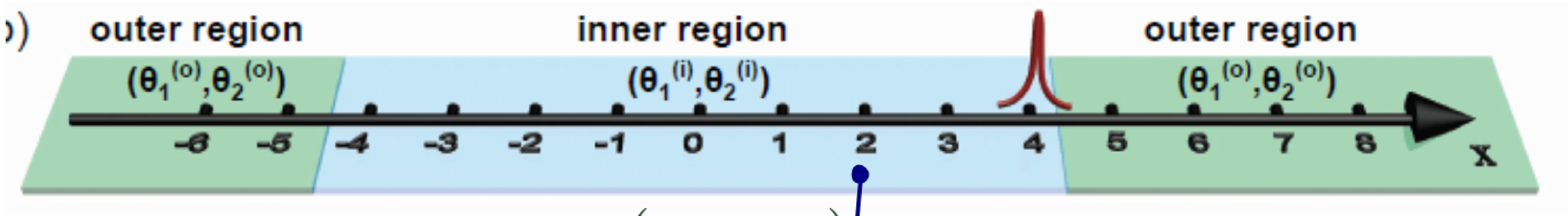
$$U_{ll} = L' S C(\theta_2) L S C(\theta_1)$$

- Shifting origin of imaginary energy:

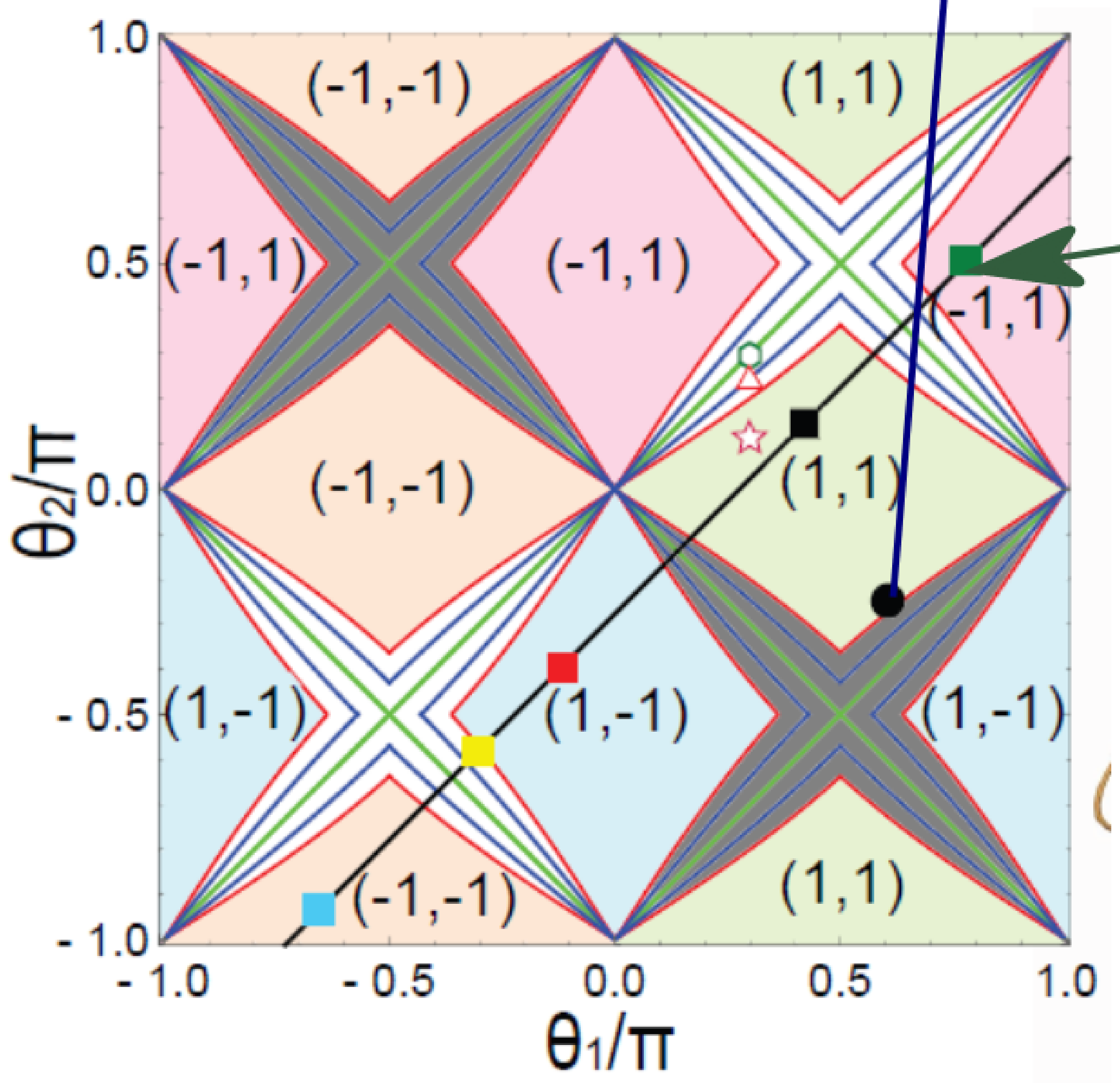
$$e^{\varepsilon_0} U_{ll} = U_{gl}$$

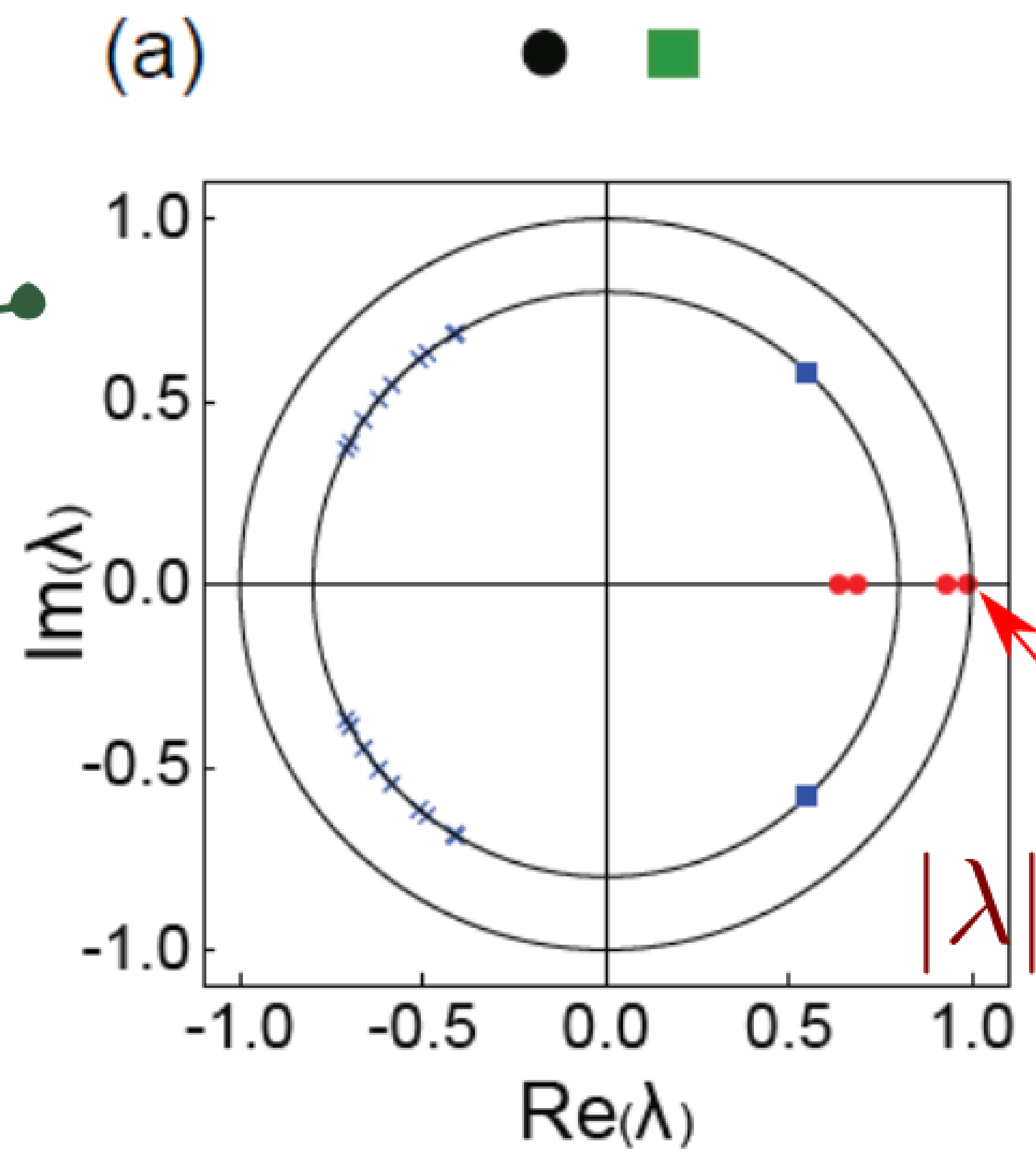
Experimental result : edge states





Topological numbers (ν_0, ν_π):



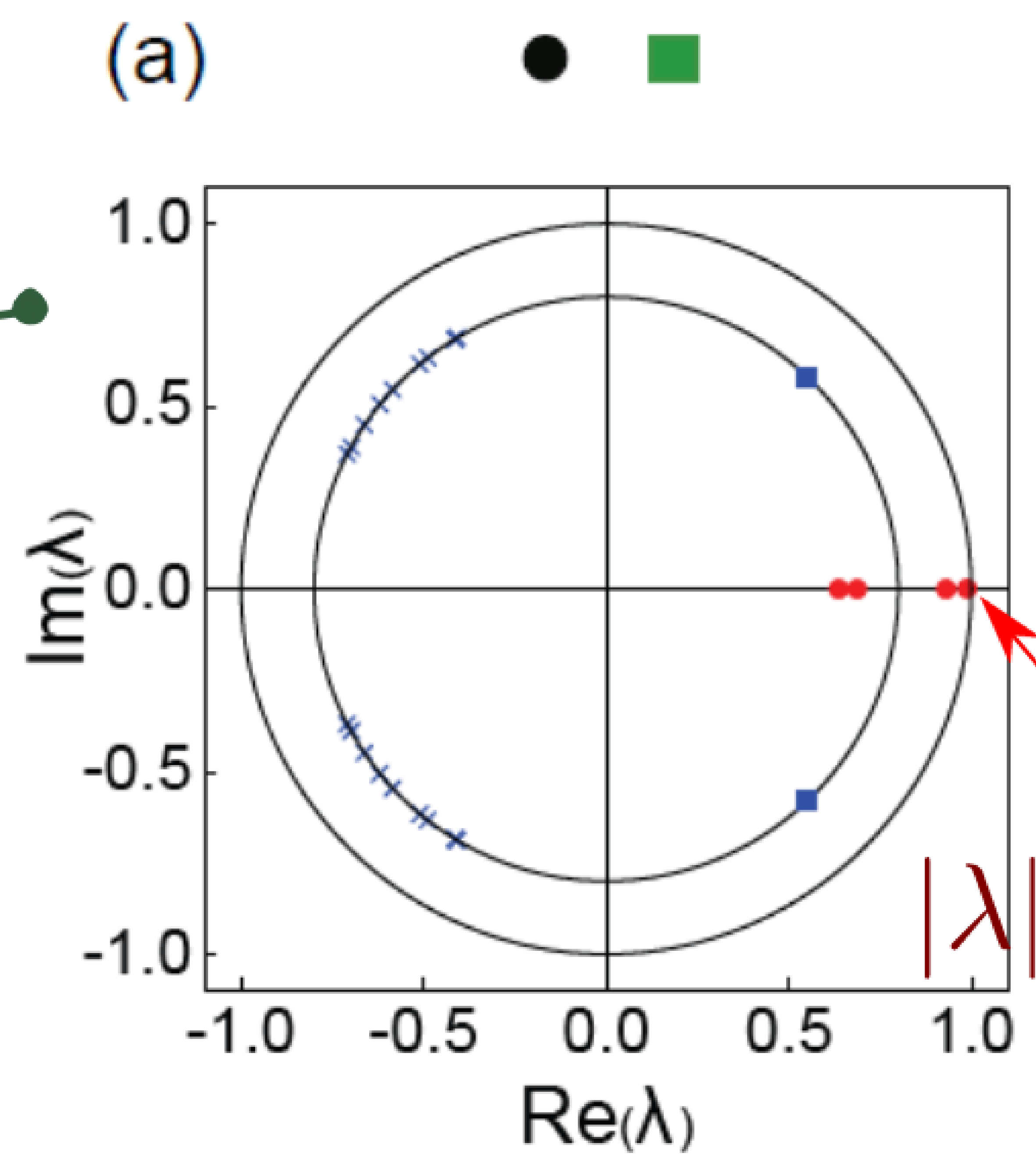


$|\lambda| = 0.9893\ldots$

≈ 1

P

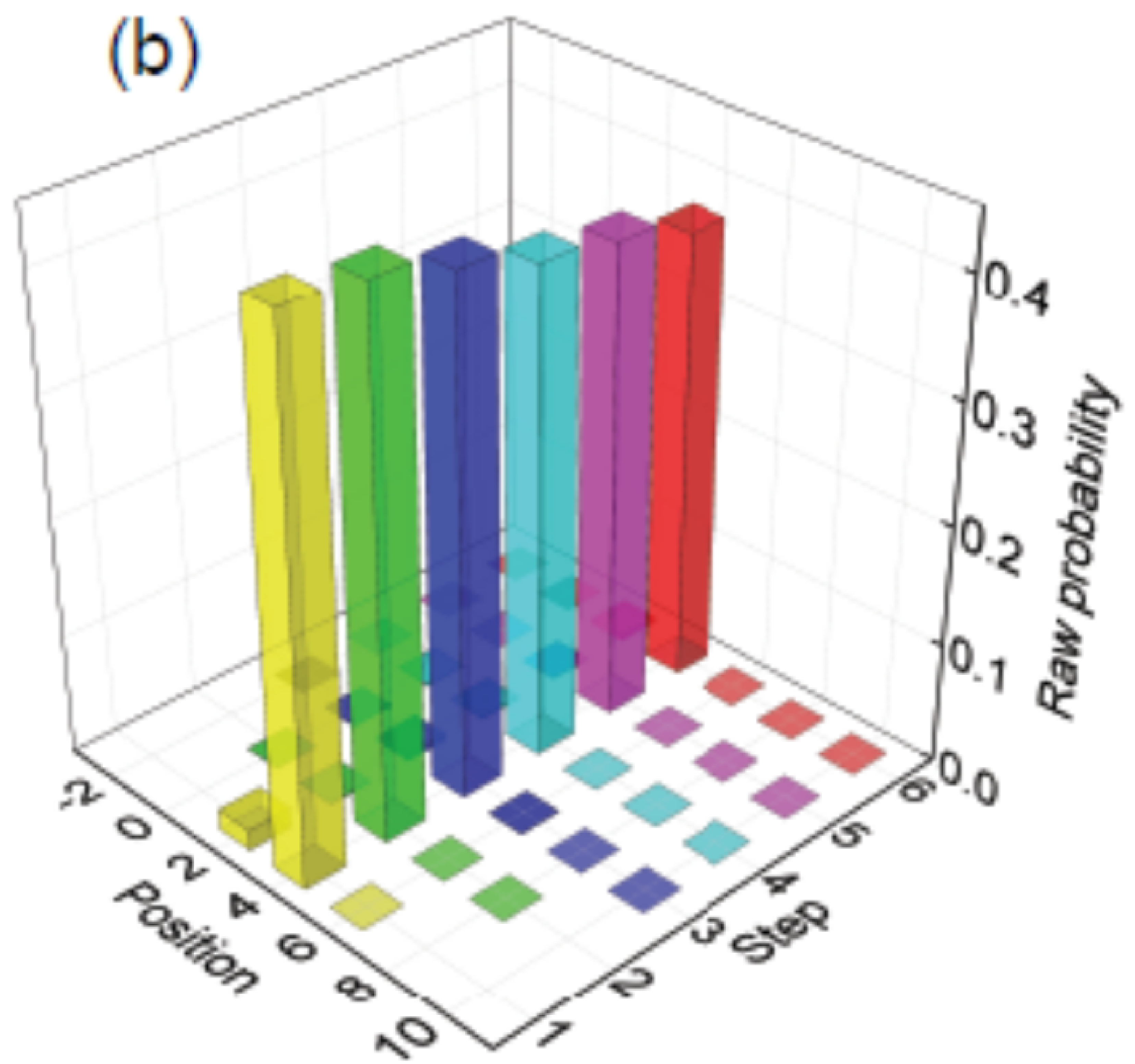
Survival prob. for bulk states : 3%

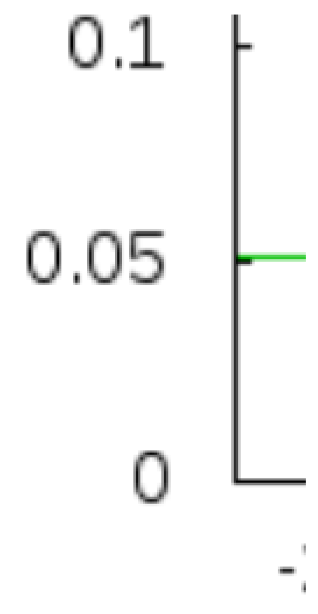


$|\lambda| = 0.9893...$

≈ 1

$P_{\mathrm{raw}}(x,t)$





Survival prob. for bulk states : 3%

$$l^{2 \cdot 6}/2 = 0.8^{2 \cdot 6}/2 \approx 0.03$$

The higher prob. at x=4 originates from $\mathcal{PT}$ symmetry breaking of edge states.

Summary

- トポロジカル絶縁体の基本的な知識を使い、量子ウォークのエッジ状態を誘起できる.

- 開放量子系における現象を、 $\mathcal{PT}$ 対称性の観点から理解・制御できる

- トポロジカル相の動的制御により、量子状態輸送が可能

PRB 84, 195139 (2011).
PRB 88, 121406(R) (2013).
PRB 92, 045424 (2015).

PRA 93, 062116 (2016).
IIS 23, 95 (2017) [arXiv:1608.00719]
arXiv:1609.09650.
Nature Physics 13, 1117 (2017)