Time-Energy Uncertainty Relation for Quantum Measurements

Takayuki Miyadera

T.M., Found. Phys. (2016),
T.M. unpublished,
K.Ito and T.M. arXiv:1711.02322

Kashiwa, August, 2018

Time-Energy Uncertainty Relations

Long history and debates.....

3

The Time–Energy Uncertainty Relation *

Paul Busch

Department of Mathematics, University of York, York, UK

3.1 Introduction

The time–energy uncertainty relation

$$\Delta T \, \Delta E \geq \frac{1}{2}\hbar \tag{3.1}$$

has been a controversial issue since the advent of quantum theory, with respect to appropriate formalisation, validity and possible meanings. Already the first formulations due to Bohr, Heisenberg, Pauli and Schrödinger are very different, as are the interpretations of the terms used. A comprehensive account of the development of this subject up to the 1980s is provided by a combination of the reviews of Jammer [1], Bauer and Mello [2], and Busch [3, 4]. More recent reviews are concerned with different specific aspects of the subject: [5, 6, 7]. The purpose of this chapter is to show that different types of time energy uncertainty relation can indeed be deduced in specific contexts, but that there is no unique universal relation that could stand on equal footing with the position–momentum uncertainty relation. To this end, we will survey the various formulations of a time energy uncertainty relation, with a brief assessment of their validity, and along the way we will indicate some new developments that emerged since the 1990s (Sects. 3.3,3.4, and 3.6). In view of the existing reviews, references to older work will be restricted to a few key sources. A distinction of three aspects of time in quantum theory introduced in [3] will serve as a guide for a systematic classification of the different approaches (Sect. 3.2).

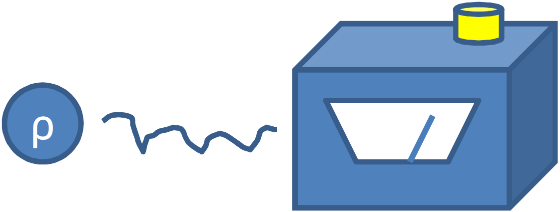
* Revised version of Chapter 3 of the 2nd edition of the Monograph *Time in Quantum Mechanics*, eds. G. Muga *et al*, Springer-Verlag, forthcoming 2007.

1 Introduction, 2. The Threefold Role of Time in Quantum Theory, 3. Relation between External Time and Energy Spread, 4. Relation Involving Intrinsic Time, 5. Quantum Clock, 6. Relations Based on Time Observables, 7. Conclusions

Main Problem

Measurement process is an interaction between System and Apparatus

$$H = H_S + V + H_A$$



How large energy is needed for a quantum measurement?

Possible Answer

Any limitation on τE_A ?

τ :: measurement time duration

Possible Answer

Any limitation on τE_A ? τ :: measurement time duration

NONE!

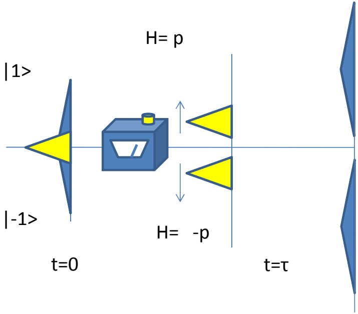
Possible Answer

Any limitation on τE_A ? τ :: measurement time duration

NONE!

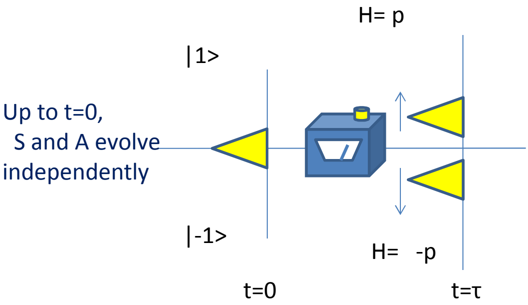
$\mathcal{H}_S = \mathbb{C}^2$: Spin 1/2, $\mathcal{H}_A = L^2(\mathbb{R})$: one particle

$$H = H_S + V + H_A = O + \sigma_z \otimes p + O$$



Sharp initial state of the apparatus makes a measurement time arbitrarily

It is OK... but who switches on the interaction?

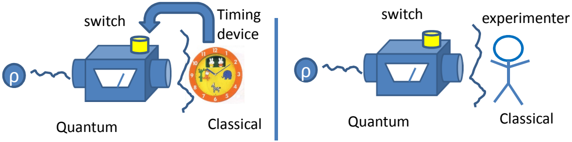


$t \leq 0 \qquad H = H_S + H_A = O \qquad\qquad t \geq 0 \qquad H = V$

This model does not describe a mechanism to switch on the interaction.

Border between Quantum and Classical

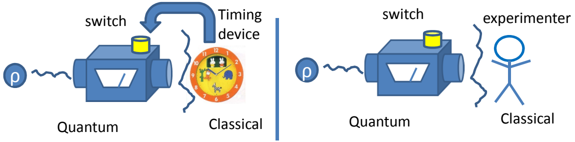
Border (Interface) = Heisenberg cut
Rough idea:



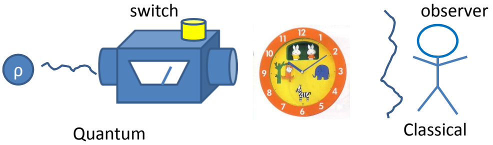
The timing device that switches on the interaction is in classical side.

Border between Quantum and Classical

Border (Interface) = Heisenberg cut
Rough idea:



The timing device that switches on the interaction is in classical side.



What if we include a switching on mechanism in the quantum side?

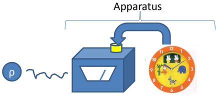
cf.) M.P. Woods, R.Silva, J. Oppenheim (2016), H. Tajima, N. Shiraishi, K. Saito (2017).

How can we formulate the quantum “switching-on” process?

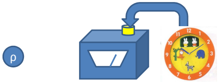
Is there any nontrivial bound on energy of the “enlarged” apparatus?

A Formulation of Switching-on process

Condition 0: The total system (= system $\mathcal{H}_S$ + apparatus $\mathcal{H}_A$) is described quantum mechanically and the dynamics is governed by a total Hamiltonian $H = H_S + H_A + V$.



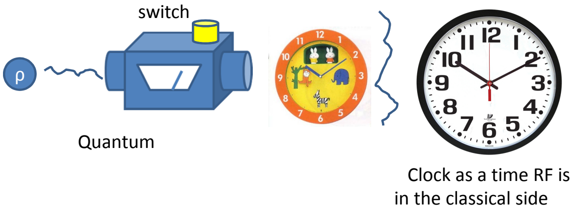
Condition1: Up to time $t = 0$, the state evolves as if there is no interaction (as if each system is isolated).



A Formulation of Switching-on process

Condition1: Up to time $t = 0$, the state evolves as if there is no interaction (as if each system is isolated).

Note: We do not shift the border too much! (cf: quantum time)



The time t is an external classical parameter. Thus the time evolution is described by the Schrödinger equation.

A Formulation of Switching-on process

(contd.)

If each system is isolated, the states evolve as,

$$\rho(t) = e^{-\frac{H_S t}{\hbar}} \rho e^{i\frac{H_S t}{\hbar}} \qquad \textit{System}$$

$$\sigma(t) = e^{-i\frac{H_A t}{\hbar}} \sigma(0) e^{i\frac{H_A t}{\hbar}} \qquad \textit{Apparatus}$$

Thus $\sigma(0)$, H_A and V must satisfy the following:

> For any state ρ of the system, for any $t \leq 0$ it holds that
>
> $$e^{-i\frac{Ht}{\hbar}} (\rho \otimes \sigma(0)) e^{i\frac{Ht}{\hbar}} = \rho(t) \otimes \sigma(t),$$
>
> where $H = H_S + H_A + V$.

Or equivalently,

For any state ρ of the system, it holds that for any $t \leq 0$,

$$[V, \rho(t) \otimes \sigma(t)] = 0.$$

A Formulation of Switching-on process

Condition 0:
The total system (= system + apparatus) is described as a closed quantum system.

Condition 1:
For any state ρ of the system, for any $t \leq 0$ it holds that

$$e^{-i\frac{Ht}{\hbar}}(\rho \otimes \sigma(0))e^{i\frac{Ht}{\hbar}} = \rho(t) \otimes \sigma(t),$$

where $H = H_S + H_A + V$.

Condition 2: (Interaction must be nontrivial)
There exists a state ρ and a time $t > 0$ such that

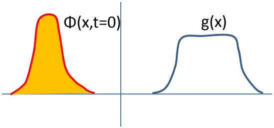
$$[V, \rho(t) \otimes \sigma(t)] \neq 0.$$

Example

$\mathcal{H}_S = \mathbb{C}^2$ spin $1/2$, $\mathcal{H}_A = L^2(\mathbb{R})$ one-particle

$$H = H_S + V + H_A = O + \sigma_z \otimes g(q) + \mathbf{1} \otimes p.$$

Supports of $\phi(x, t=0) = \langle x|\phi(t=0)\rangle$ and $g(x)$ satisfy:
$supp\phi(x,0) \subset (-\delta, 0)$, $suppg(x) \subset (0, \Delta)$.



For $t \geq \tau = \delta + \Delta$, an initial state $|\Omega\rangle = \frac{1}{\sqrt{2}}(|1\rangle + |-1\rangle)$ evolves as,

$$\rho(t) = \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1| + e^{-\frac{2i}{\hbar}\int_0^\Delta dx g(x)}|0\rangle\langle 1| + e^{\frac{2i}{\hbar}\int_0^\Delta dx g(x)}|1\rangle\langle 0|\right)$$

$\neq \rho^0(t)$ state with V=0

H is unbounded

Theorem 1

Condition 0, 1, 2 $\Rightarrow H = H_S + V + H_A$ *is two-side unbounded*

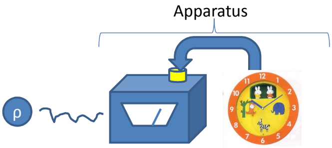
Proof.

Assume that H is lower bounded. For an arbitrary $|\Psi\rangle \in \mathcal{H}_S \otimes \mathcal{H}_A$ and an arbitrary state $|\Omega\rangle \in \mathcal{H}_S$ we define

$$f_{\Psi,\Omega}(t) := \langle\Psi|Ve^{-i\frac{H}{\hbar}t}|\Omega \otimes \phi(0)\rangle.$$

Due to Condition 1, this is vanishing for $t \leq 0$. As H is lower bounded, $f_{\Psi,\Omega}(z) := \langle\Psi|Ve^{-i\frac{H_c}{\hbar}z}|\Omega \otimes \phi(0)\rangle$ can be defined for $Im(z) \leq 0$ and is analytic for $Im(z) < 0$. The Schwarz reflection principle concludes that $f_{\Psi,\Omega}$ can be extended to an analytic function on $\mathbb{C} \setminus \{s|s > 0\}$. Because $f_{\Psi,\Omega}(z) = 0$ on $z \in \mathbb{C} \setminus \{s|s > 0\}$, the continuity shows that $f_{\Psi,\Omega}(t) = 0$ for $t \in \mathbb{R}$. That is, $Ve^{-i\frac{H}{\hbar}t}|\Omega \otimes \phi(0)\rangle = 0$. □

Energy fluctuation required for quantum measurements



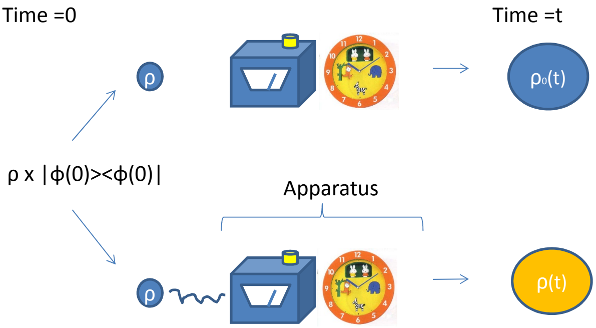
$$H = H_S + V + H_A$$

Observation 1. (Time-energy tradeoff)
Large energy fluctuation of an apparatus is needed to disturb system states to some extent in short time duration.

Observation 2. (Information-disturbance tradeoff)
Measurement process destroys states.

Time-energy tradeoff

We compare two states.



States of the system at time t:

$\rho^0(t)$: Without interaction

$\rho(t)$: With interaction

Time-energy tradeoff

Theorem 2

Condition 1 $\Rightarrow$ *For* $t \leq \frac{\pi\hbar}{2\Delta H_A}$,

$$\cos\left(\frac{\Delta H_A t}{\hbar}\right) \leq F(\rho(t), \rho^0(t)).$$

$$\Delta H_A = \sqrt{\langle\phi(0)|H_A^2|\phi(0)\rangle - \langle\phi(0)|H_A|\phi(0)\rangle^2}$$

Energy fluctuation of apparatus

$$F(\sigma_1, \sigma_2) = \mathrm{tr}[\sqrt{\sqrt{\sigma_1}\sigma_2\sqrt{\sigma_1}}]$$

Fidelity $0 \leq F \leq 1$

To give rise to strong disturbance, large energy fluctuation is required.

Time-energy tradeoff

Proof.

We introduce a "time-delayed" apparatus.
The dynamics is governed by $H = H_S + H_A + V$.

$\Theta_t(0) = \rho \otimes |\phi(-t)\rangle\langle\phi(-t)| \quad \mapsto \quad \Theta_t(t) = \rho^0(t) \otimes |\phi(0)\rangle\langle\phi(0)|$

v.s.

$\Theta_0(0) = \rho \otimes |\phi(0\rangle\langle\phi(0)| \quad \mapsto \quad \Theta_0(t),\ \mathrm{tr}_A[\Theta_0(t)] = \rho(t)$.

$F(\Theta_t(0), \Theta_0(0)) = F(\Theta_t(t), \Theta_0(t)$ fidelity is unitary invariant

$|\langle\phi(-t)|\phi(0)\rangle| \leq F(\rho^0(t), \rho(t))$ fidelity under partial trace

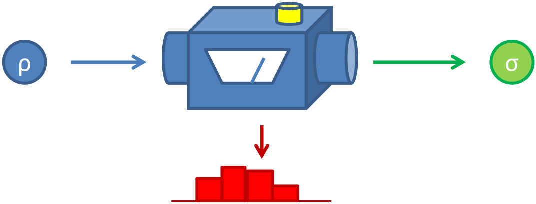
$|\langle\phi(-t)|\phi(0)\rangle| \geq \cos\left(\frac{\Delta H_A t}{\hbar}\right)$ Mandelstam-Tamm u. r.

$$\cos\left(\frac{\Delta H_A t}{\hbar}\right) \leq F(\rho^0(t), \rho(t))$$

□

Information-disturbance tradeoff

Incompatibility



Information gain causes inevitable disturbance of system states
A lot of quantitative formulations have been obtained thus far...

Information-disturbance tradeoff

Lemma 3

Suppose that an interaction between a system and an apparatus describes a measurement process of a Projection-Valued Measure $\mathrm{P} = \{\mathrm{P}_n\}$*. Then there exists a state* ρ *of the system satisfying* $F(\rho(\tau), \rho^0(\tau)) \leq \frac{1}{\sqrt{2}}$.

Proof.

There is a pair of states: $\{|0\rangle, |1\rangle\}$ s.t.,
$P_0|0\rangle = |0\rangle$, $P_1|0\rangle = 0$, $P_1|1\rangle = |1\rangle$, $P_0|1\rangle = 0$.
This pair is perfectly distinguished.

$\Rightarrow$ Another pair $\{|+\rangle, |-\rangle\}$ $(|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle))$. is completely destroyed.
$|\pm\rangle\langle\pm| \mapsto |\pm'\rangle\langle\pm'| = \rho^0_{\pm}(\tau)$ (Orthogonal states: unitarity)
$|\pm\rangle\langle\pm| \mapsto \rho(\tau)$ (Identical state)

$$F(\rho(\tau), |+'\rangle\langle+'|)^2 + F(\rho(\tau), |-'\rangle\langle-'|)^2 \leq \mathrm{tr}[\rho(\tau)] = 1.$$

$$\min\{F(\rho(\tau), |+'\rangle\langle+'|), F(\rho(\tau), |-'\rangle\langle-'|)\} \leq \frac{1}{\sqrt{2}}.$$

□

Energy required for quantum measurements

Theorem 4

For a physical system to describe a measurement process of a sharp observable, energy fluctuation ΔH_A of an apparatus and measurement time duration τ must satisfy,

$$\Delta H_A \cdot \tau \geq \frac{\pi \hbar}{4}.$$

It follows just by combining Theorem 2 and Lemma 3.

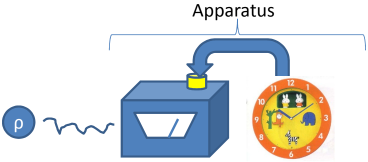
$$\cos\left(\frac{\Delta H_A t}{\hbar}\right) \leq F(\rho(t), \rho^0(t)).$$

$$\min\{F(\rho(t), |+'\rangle\langle +'|), F(\rho(t), |-'\rangle\langle -'|)\} \leq \frac{1}{\sqrt{2}}.$$

Main Problem

Measurement process is an interaction between System and Apparatus

$$H = H_S + V + H_A$$



How large energy is needed for a quantum measurement?

$$\Delta H_A \cdot \tau \geq \frac{\pi\hbar}{4}$$

For an observable with infinite outcomes, it holds that $\Delta H_A \cdot \tau \geq \frac{\pi\hbar}{2}$.

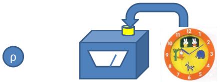
More physical condition

For a model to satisfy the conditions strictly, Hamiltonian must be two-sided unbounded.

Too strict = Unphysical!

The conditions should be weakened.

Condition1': Up to time $t = 0$, the state evolves as if there is almost no interaction (as if each system is almost isolated).



More physical condition: a possible formulation

If there is no interaction between the system and the apparatus, the state evolves independently:

$$e^{-i\frac{H_0 t}{\hbar}}|\Omega_S \otimes \phi_A\rangle = |\Omega_S(t)\rangle \otimes |\phi_A(t)\rangle.$$

In particular, if we "prepare" the state at time $t = -T \ll 0$, the prepared state must be

$$e^{i\frac{H_0 T}{\hbar}}|\Omega_S \otimes \phi_A\rangle = |\Omega_S(-T)\rangle \otimes |\phi_A(-T)\rangle.$$

But in reality interaction exists and the total Hamiltonian is $H = H_0 + V$. So the state prepared at $-T$ evolves as, at time t,

$$e^{-i\frac{Ht}{\hbar}}e^{-i\frac{HT}{\hbar}}|\Omega_S(-T)\rangle \otimes |\phi_A(-T)\rangle = e^{-i\frac{Ht}{\hbar}}(e^{-i\frac{HT}{\hbar}}e^{i\frac{H_0 T}{\hbar}})|\Omega_S \otimes \phi_A\rangle.$$

More physical condition: a possible formulation

Real state at time t (prepared at time $-T$) :

$$e^{-i\frac{Ht}{\hbar}}(e^{-i\frac{HT}{\hbar}}e^{i\frac{H_0 T}{\hbar}})|\Omega_S \otimes \phi_A\rangle$$

As "preparation" by an external agent makes the total system not closed. Therefore we remove this preparation procedure to the infinite past:

Real state at time t:

$$e^{-i\frac{Ht}{\hbar}}\lim_{T\to\infty}(e^{-i\frac{HT}{\hbar}}e^{i\frac{H_0 T}{\hbar}})|\Omega_S \otimes \phi_A\rangle =: e^{-i\frac{Ht}{\hbar}}W|\Omega_S \otimes \phi_A\rangle,$$

where $W := \lim_{T\to\infty} e^{-i\frac{HT}{\hbar}}e^{\frac{iH_0 T}{\hbar}}$ is a wave operator in the scattering theory.

A scattering theory formulation

Condition 1':
There exists $\epsilon > 0$ such that the following holds. For any state $|\Omega_S\rangle$ of the system, for any $t \leq 0$ it holds that

$$\| e^{-i\frac{Ht}{\hbar}} W |\Omega_S \otimes \phi_A\rangle - e^{-i\frac{H_0 t}{\hbar}} |\Omega_S \otimes \phi_A\rangle \| \leq \epsilon.$$

Example: A particle on a two-dimensional space

$$H = H_S \otimes \mathbf{1} + \mathbf{1} \otimes \frac{\mathbf{p}^2}{2m} + X \otimes v(q),$$

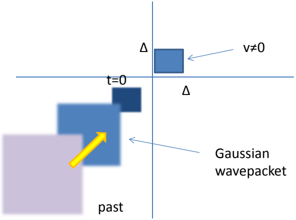
where *supp* $v \subset [0, \Delta] \times [0, \Delta]$. $\phi_A(x, y)$ is a product of Gaussian wave functions centered at $(-x_0, -x_0)$ with variant σ^2 for each coordinate and group velocity (v_g, v_g).

A scattering theory formulation

Example: A particle on a two-dimensional space

$$H = H_S \otimes \mathbf{1} + \mathbf{1} \otimes \frac{\mathbf{p}^2}{2m} + X \otimes v(\mathbf{q}),$$

where *supp* $v \subset [0, \Delta] \times [0, \Delta]$. $\phi_A(x, y)$ is a product of Gaussian wave functions centered at $(-x_0, -x_0)$ with variant σ^2 for each coordinate and group velocity (v_g, v_g).



For sufficiently large x_0 and v_g, or for sufficiently small σ, Condition 1' is satisfied: $\| e^{-i\frac{Ht}{\hbar}} W |\Omega_S \otimes \phi_A\rangle - e^{-i\frac{H_0 t}{\hbar}} |\Omega_S \otimes \phi_A\rangle \| \leq \epsilon$.

A scattering theory formulation

Real scalar massless field

$$H = \mathbf{1} \otimes \frac{1}{2} \int d\mathbf{x} \left(: (\partial_0 \phi(\mathbf{x}))^2 : + : (\nabla \phi(\mathbf{x}))^2 : \right) + \frac{1}{2} P \otimes \int d\mathbf{x} V(\mathbf{x}) : \phi(\mathbf{x})^2 :,$$

where $P = |1\rangle\langle 1|$ (an observable to be measured). *suppV* is compact.

Sector $P = 0$:

$$H = \frac{1}{2} \int d\mathbf{p} a^*(\mathbf{p}) a(\mathbf{p}), \qquad \langle \mathbf{x} | \mathbf{p} \rangle = \frac{1}{(2\pi)^{3/2}} e^{i\mathbf{p}\cdot\mathbf{x}}.$$

Sector $P = 1$:

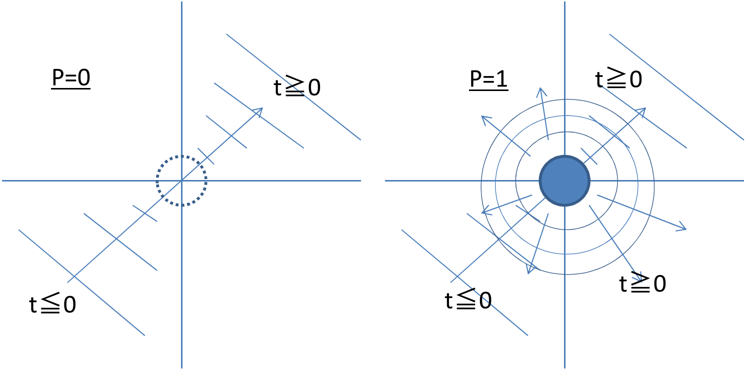
$$H = \frac{1}{2} \int d\mathbf{p} \tilde{a}^*(\mathbf{p}) \tilde{a}(\mathbf{p}),$$

where $\tilde{a}(\mathbf{p})|\Omega\rangle = |\tilde{\mathbf{p}}\rangle$ is a scattering state

$$(-\Delta + V(\mathbf{x}))\langle \mathbf{x} | \tilde{\mathbf{p}} \rangle = |\mathbf{p}| \langle \mathbf{x} | \tilde{\mathbf{p}} \rangle,$$

$$\langle \mathbf{x} | \tilde{\mathbf{p}} \rangle = \langle \mathbf{x} | \mathbf{p} \rangle + spherical wave$$

A scattering theory formulation



Condition 1':
There exists $\epsilon > 0$ such that the following holds. For any state $|\Omega_S\rangle$ of the system, for any $t \leq 0$ it holds that

$$\| e^{-i\frac{Ht}{\hbar}} W |\Omega_S \otimes \phi_A\rangle - e^{-i\frac{H_0 t}{\hbar}} |\Omega_S \otimes \phi_A\rangle \| \leq \epsilon.$$

A scattering theory formulation

$\mathbf{P} = \{P_n\}_n$: a PVM to be measured.
$\{E_n\}$: a pointer POVM on the apparatus.

Measurement with error δ and measurement time τ:

$$|\langle\Omega_S|P_n|\Omega_S\rangle - \langle\Omega_S\otimes\phi_A|W^* e^{i\frac{H\tau}{\hbar}}(\mathbf{1}\otimes E_n)e^{-i\frac{H\tau}{\hbar}}W|\Omega_S\otimes\phi_A\rangle| \leq \delta.$$

Theorem 5

Energy fluctuation of the apparatus
$\Delta H_A := (\langle\phi_A|H_A^2|\phi_A\rangle - \langle\phi_A|H_A|\phi_A\rangle^2)^{1/2}$ *satisfies*

$$\frac{(\Delta H_A)\cdot\tau}{\hbar} \geq Arccos\sqrt{\frac{1+6\sqrt{2\epsilon+\delta}}{2}} - Arccos(1-2\epsilon).$$

Letting $\epsilon, \delta \to 0$, we regain $(\Delta H_A)\cdot\tau \geq \frac{\pi}{4}\hbar$.

Application I: Spacetime Uncertainty Relation

Quantum effect $\Rightarrow$ breakdown of spacetime continuum (?)
cf.) string theory, quantum gravity....

Doplicher, Fredenhagen, Roberts (1999) (Originally Bronstein 1936)
To measure a local observable, one needs to concentrate huge energy at its region V. It produces a black hole which prohibits information transfer and spoils the spacetime continuum.

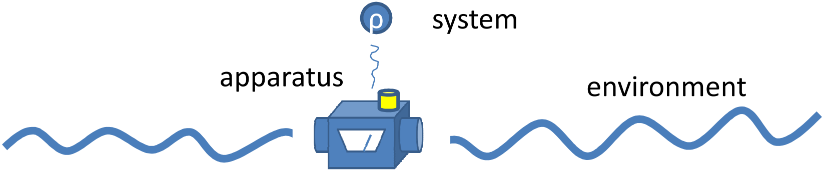
Our theorem $\Delta H_V \cdot \tau \geq \frac{\pi\hbar}{4}$ is applied.

An identification $\Delta H_V \simeq Mc^2$ leads to $Mc^2\tau \geq \frac{\pi\hbar}{4}$.
On the other hand, r, diameter of V must exceeds the Schwarzschild radius to avoid the formation of a blackhole.
$r \geq \frac{2GM}{c^2}$.
Thus we obtain $\tau \cdot r \geq \frac{\pi\hbar G}{2c^4}$.

In the paper (TM, FOOP,) a more detailed derivation is presented (using Lieb-Robinson bound).

Spacetime Uncertainty Relation

There is a loophole......

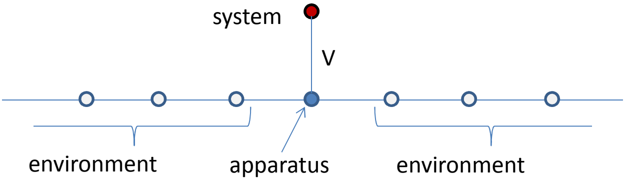


An infinite environment can show $\Delta(H_A + H_E) = \infty$.

$$\Delta(H_A + H_E)\tau \geq \frac{\pi\hbar}{4} \text{ is useless!}$$

Spacetime Uncertainty Relation

Locality of the interaction must be taken into account.
Model:



Let us consider a nearest-neighbor-interacting lattice model. The origin $0 \in \mathbf{Z}^d$ represents an apparatus.
For $\Lambda \subset \mathbf{Z}^d$, its "box Hamiltonian" is defined by

$$H_\Lambda = \sum_{x \in \Lambda} h_x + \sum_{\{x,y\} \subset \Lambda} \Phi(x, y),$$

where $\Phi(x, y) = 0$ for $d(x, y) \neq 1$.

Spacetime Uncertainty Relation

The dynamics:

System: $\gamma_t(\mathrm{A}) = e^{i\frac{H_S t}{\hbar}} \mathrm{A} e^{-i\frac{H_S t}{\hbar}}$

Apparatus-plus-Environment:

$$\beta_t(\mathrm{A}) = \lim_{\Lambda \to \mathbf{Z}^d} e^{i\frac{H_\Lambda t}{\hbar}} \mathrm{A} e^{-i\frac{H_\Lambda t}{\hbar}}.$$

Total System (without interaction): $\alpha_t^0 = \gamma_t \otimes \beta_t$

Total System (with interaction):

$$\alpha_t(\mathrm{A}) = \lim_{\Lambda} \alpha_t^\Lambda(\mathrm{A}) = \lim_{\Lambda} e^{i\frac{H_S + H_\Lambda + V}{\hbar} t} \mathrm{A} e^{-i\frac{H_S + H_\Lambda + V}{\hbar} t}.$$

No-interaction up to switching-on time: For $t \leq 0$ and an arbitrary state ρ of the system, $(\rho \otimes \omega) \circ \alpha_t = (\rho \otimes \omega) \circ \alpha_t^0$ holds.

Spacetime Uncertainty Relation

Lieb-Robinson bound:
There is a bound for information propagation speed v.
Let $\Lambda(vt)$ denote an expanding box $\Lambda(vt) := \{x|d(x,0) \leq vt\}$. For any operator A on the system, it holds that $\|\alpha_t(\mathsf{A}) - \alpha^{\Lambda(vt)}(\mathsf{A})\| \leq \epsilon$.

After some effort to bound the error, we can conclude,

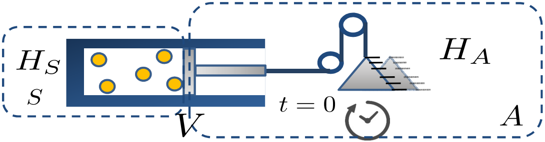
$$\Delta H_{\Lambda(v\tau)} \cdot \tau \geq \frac{\pi\hbar}{4} - \frac{\pi\hbar}{2}\sqrt{2\epsilon}.$$

Note that $\Lambda(v\tau)$ is finite.

· The no-interaction condition is generalized so that it is applicable to an infinite environment.
· The information-disturbance tradeoff works as it is relevant only to a finite system.
· Lieb-Robinson bound is used to give the "box Hamiltonian" which is sufficient to approximate the total dynamics.

Application II: Quantum Theormodynamics -Power bound

Work extraction by a quantum machine



Power = Work extraction rate:

$$P := \frac{\mathrm{tr}[\rho_S(0)H_S] - \mathrm{tr}[\rho_S(\tau)H_S]}{\tau}.$$

Theorem 6 (K.Ito, TM, arXiv:171102322)

For a process in which interaction is switched on at time 0 and off at time τ, its power is bounded as:

$$P \leq \frac{2\|H_S\|(\Delta H_A)}{\hbar}.$$

Summary

∗ We gave a formulation of "switching-on" process.

∗ Energy of an apparatus (which also has switching-on mechanism) and measurement time duration satisfies a tradeoff inequality.

∗ We applied our theorem to strengthen the argument of spacetime uncertainty relation.

∗ A universal power bound was derived.

∗ The strength of interaction V and the measurement time duration also satisfies another tradeoff inequality: $\|V\| \cdot \tau \geq \frac{\pi}{4}\hbar$

Open problems

∗ Relativistic treatment of spacetime uncertainty relation.

∗ Application to Einstein-Bohr photon box? (P. Busch)

∗ Application to thermodynamical process. (Maxwell's Demon?)

Thank you!