エンタングルメント分岐とその活用

原田健自
京都大学　大学院情報学研究科

Reference: “Entanglement branching operator”, Phys. Rev. B **97**, 045124 (2018)

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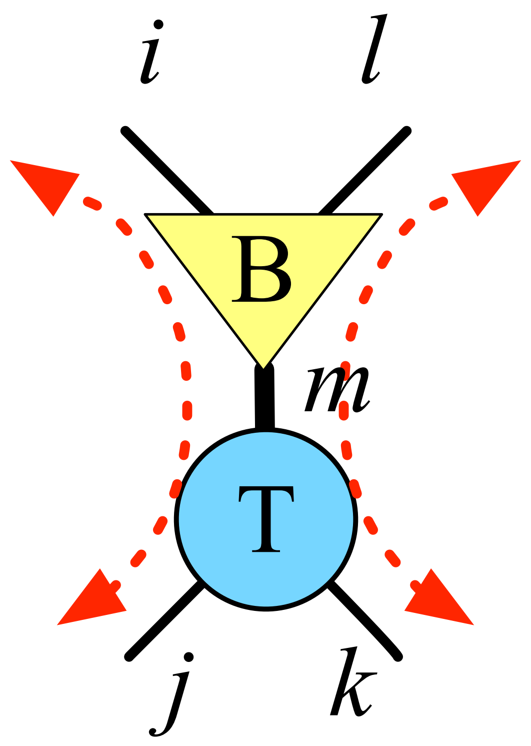

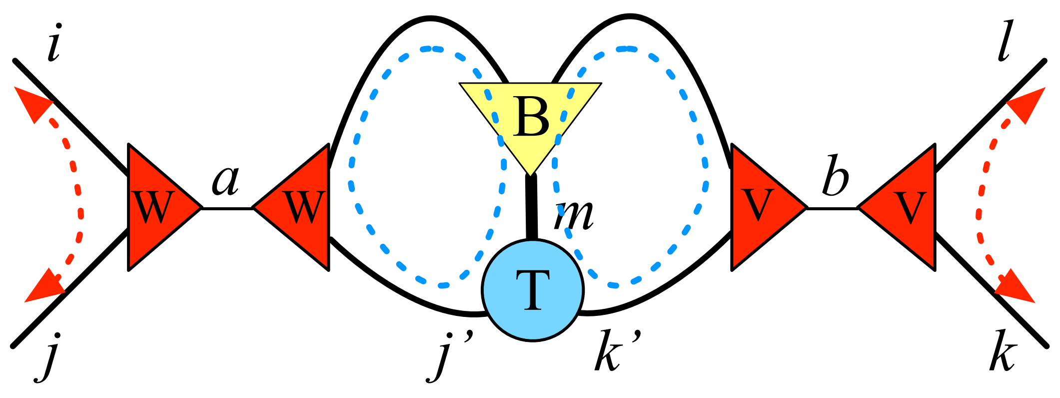

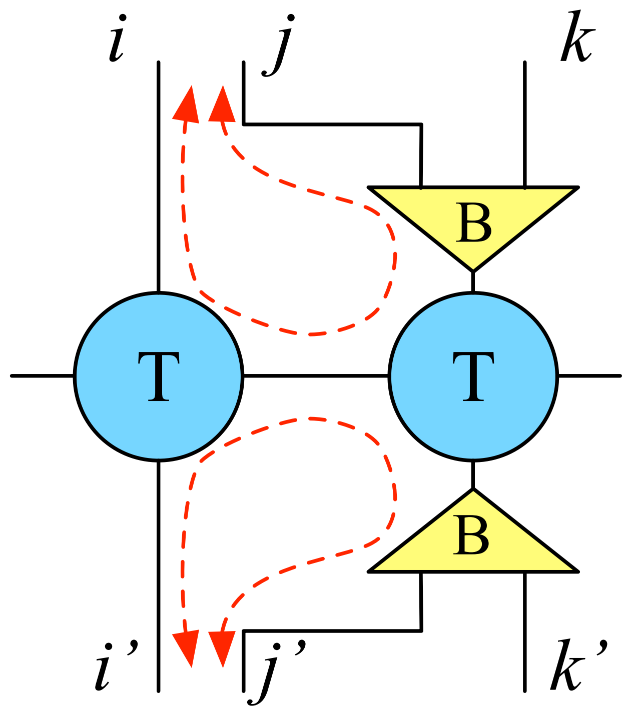

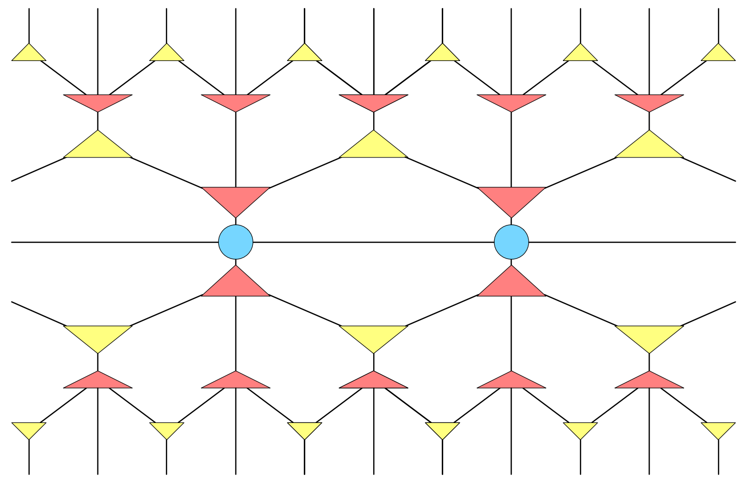

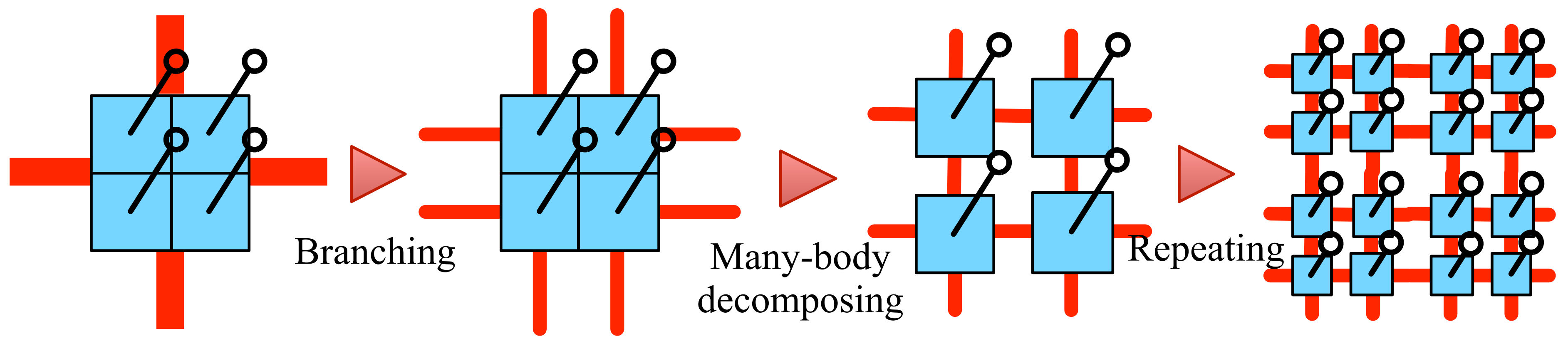

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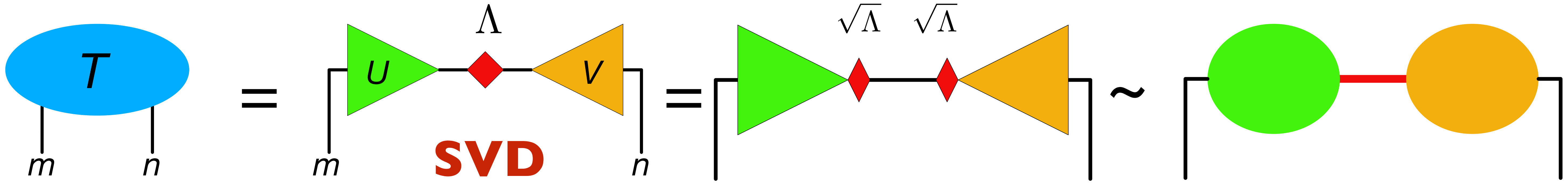
Entanglement and singular value decomposition

Schmidt decomposition

sub-system A   sub-system B

$$|\psi\rangle = \sum_{mn} (T_{mn})\ |m\rangle \otimes |n\rangle \;=\; \sum_{l=1}^{D} (\lambda_l)\ |u_l\rangle \otimes |v_l\rangle$$



Entanglement entropy $S_E(A) = -\mathrm{Tr}_{\mathrm{A}}\ \rho_A \log(\rho_A) = -\sum_{l=1}^{D} \lambda_l^2 \log(\lambda_l^2)$

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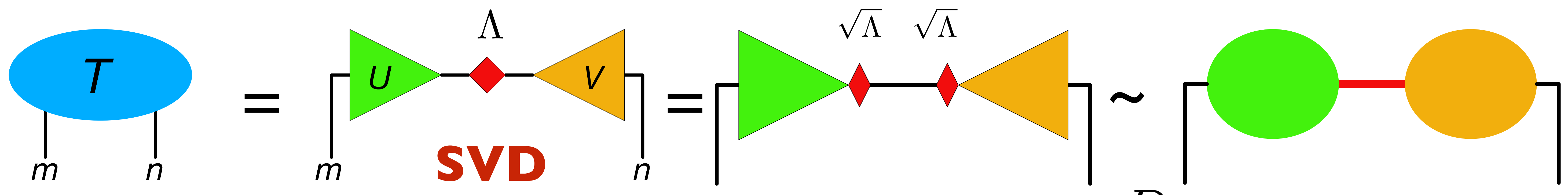

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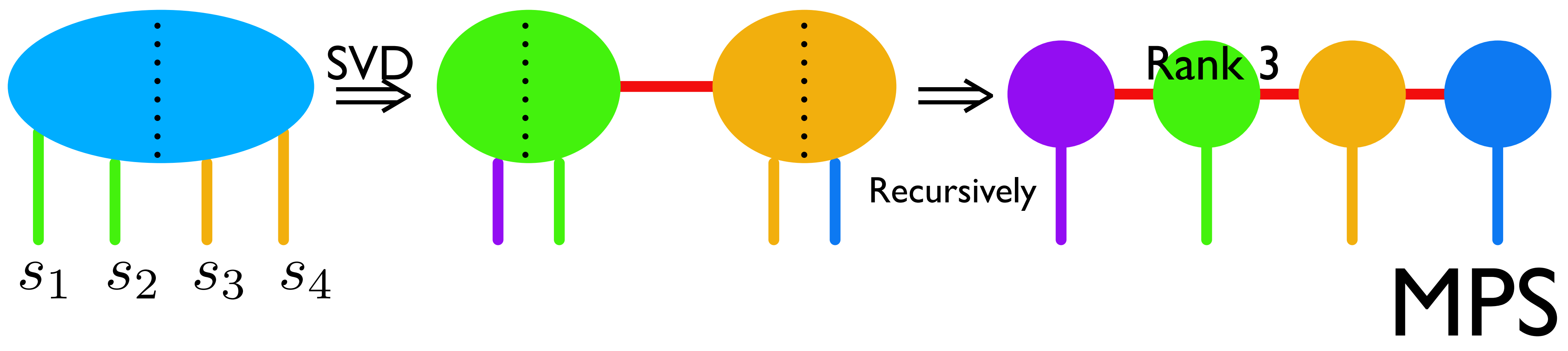
1D quantum state



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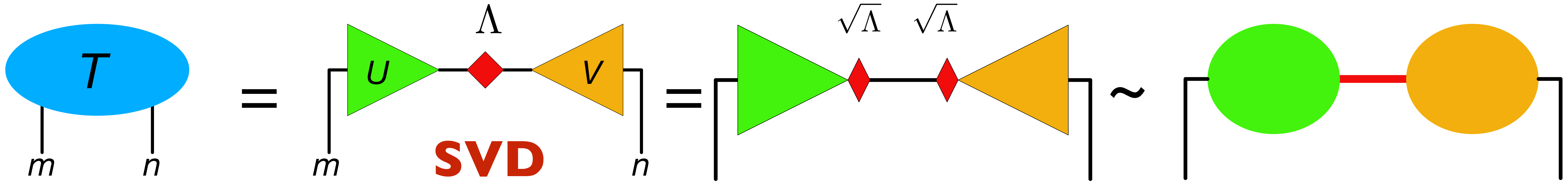

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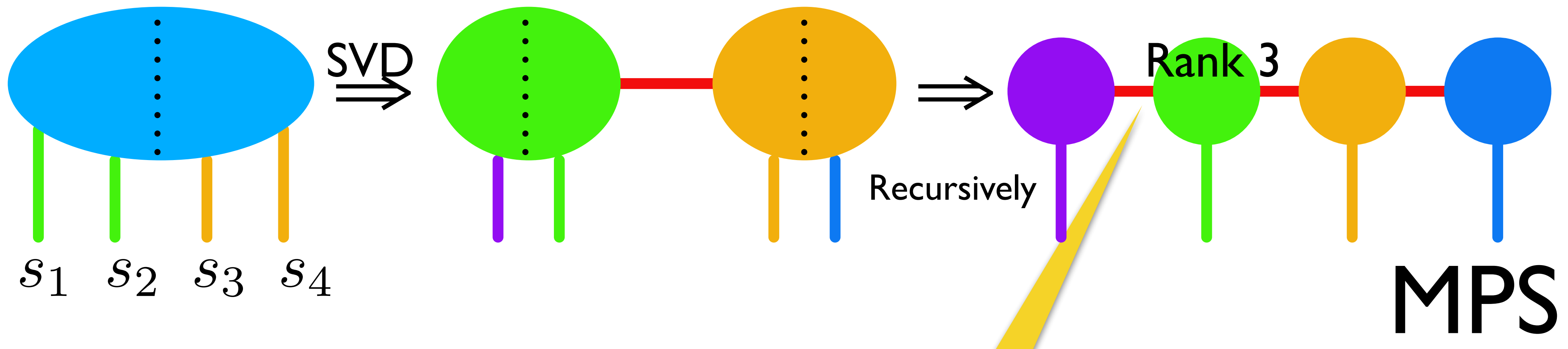
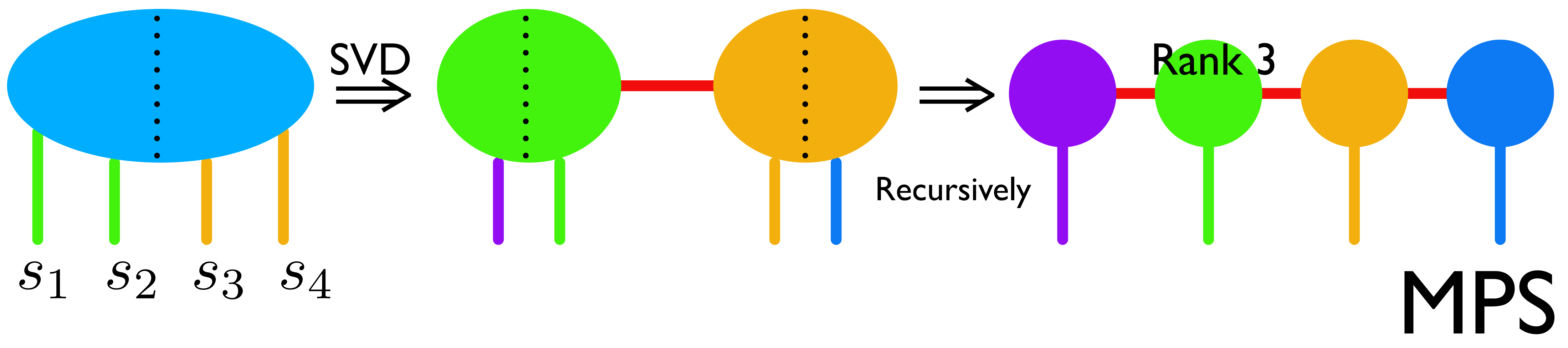


Entanglements flow on a link

Tensor network, tensor network algorithm, and entanglement flow

1D quantum state



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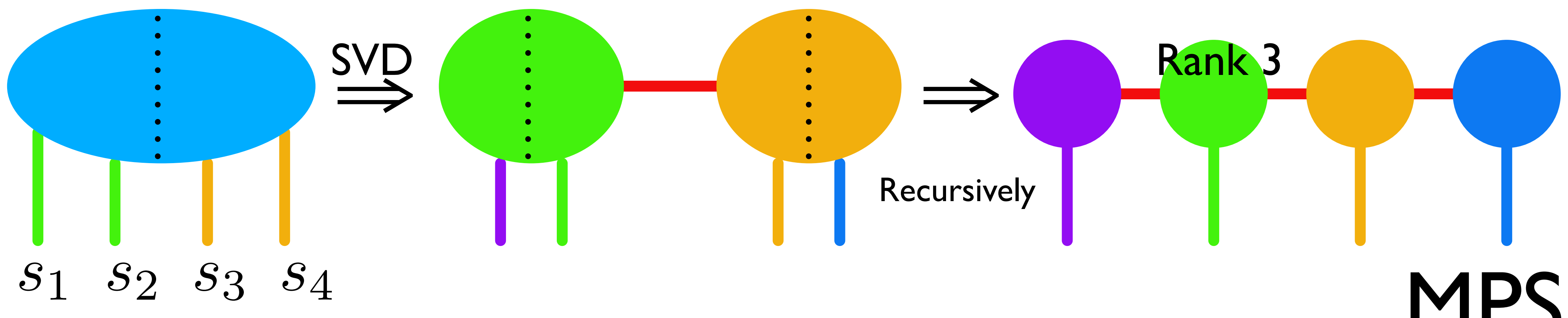
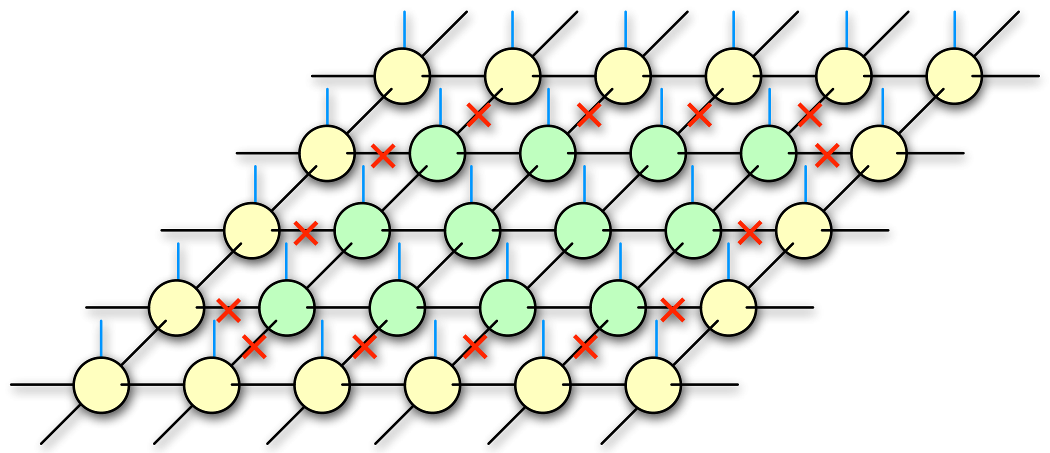


MPS

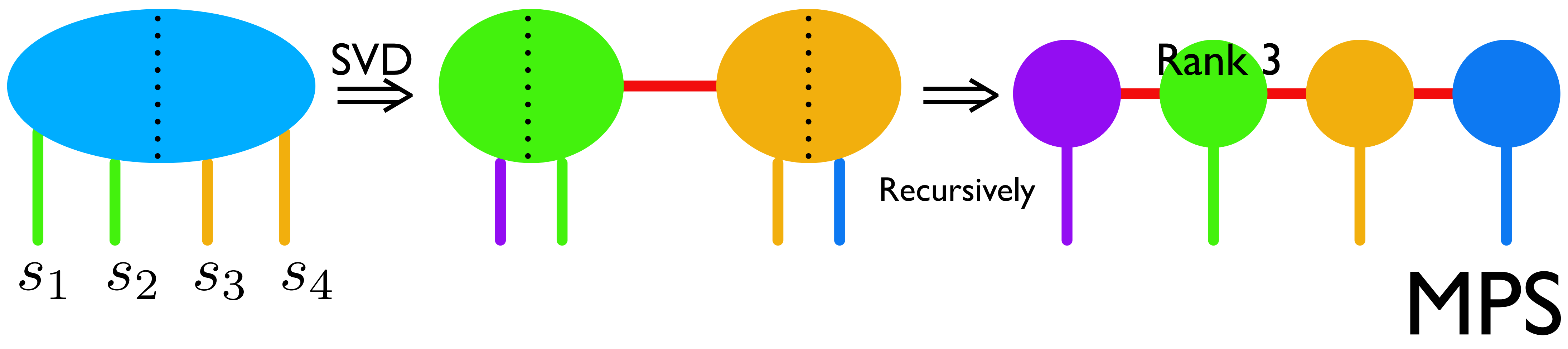
2D quantum state



PEPS(TPS)

Tensor network, tensor network algorithm, and entanglement flow

1D quantum state



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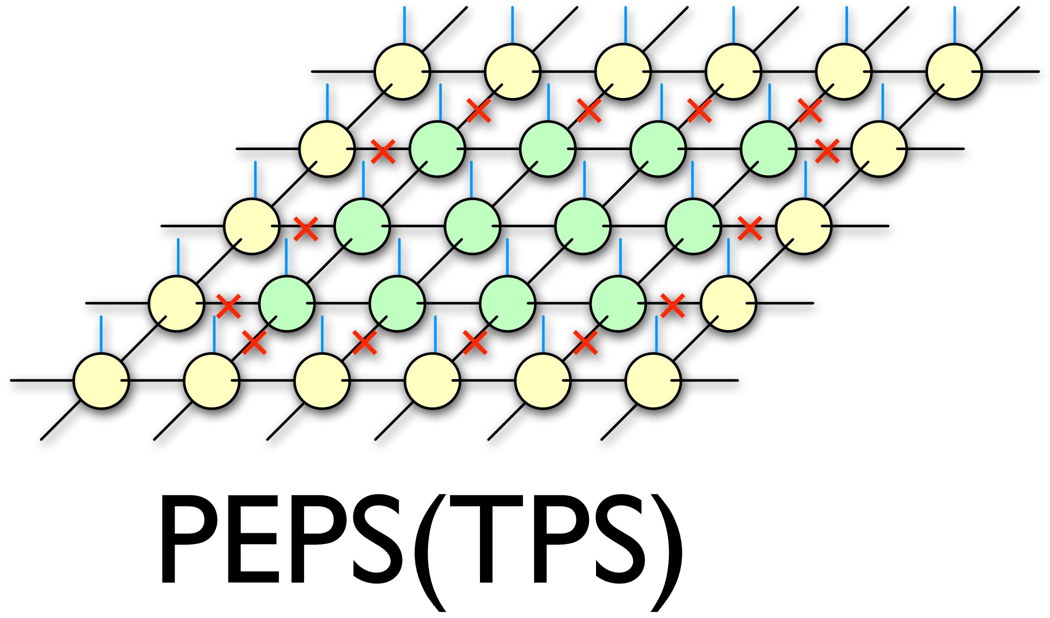
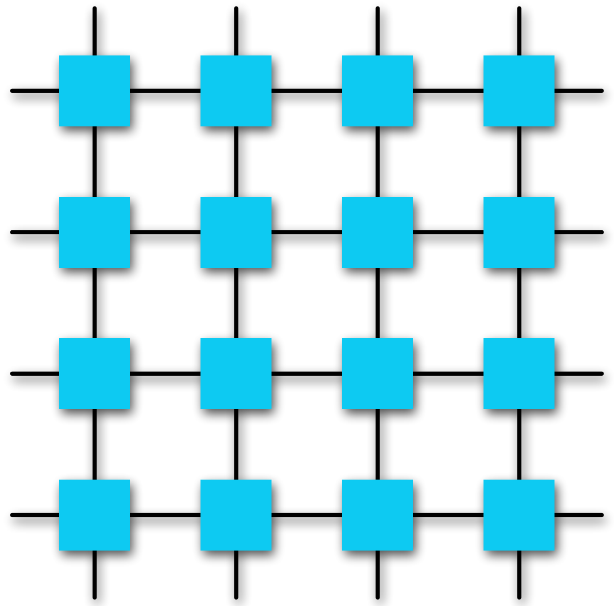


2D classical partition function



Tensor network, tensor network algorithm, and entanglement flow

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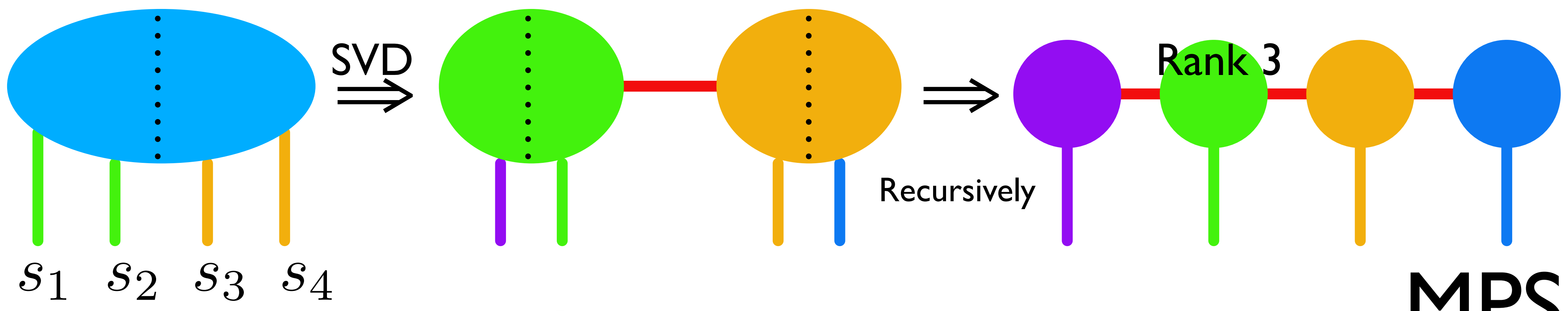
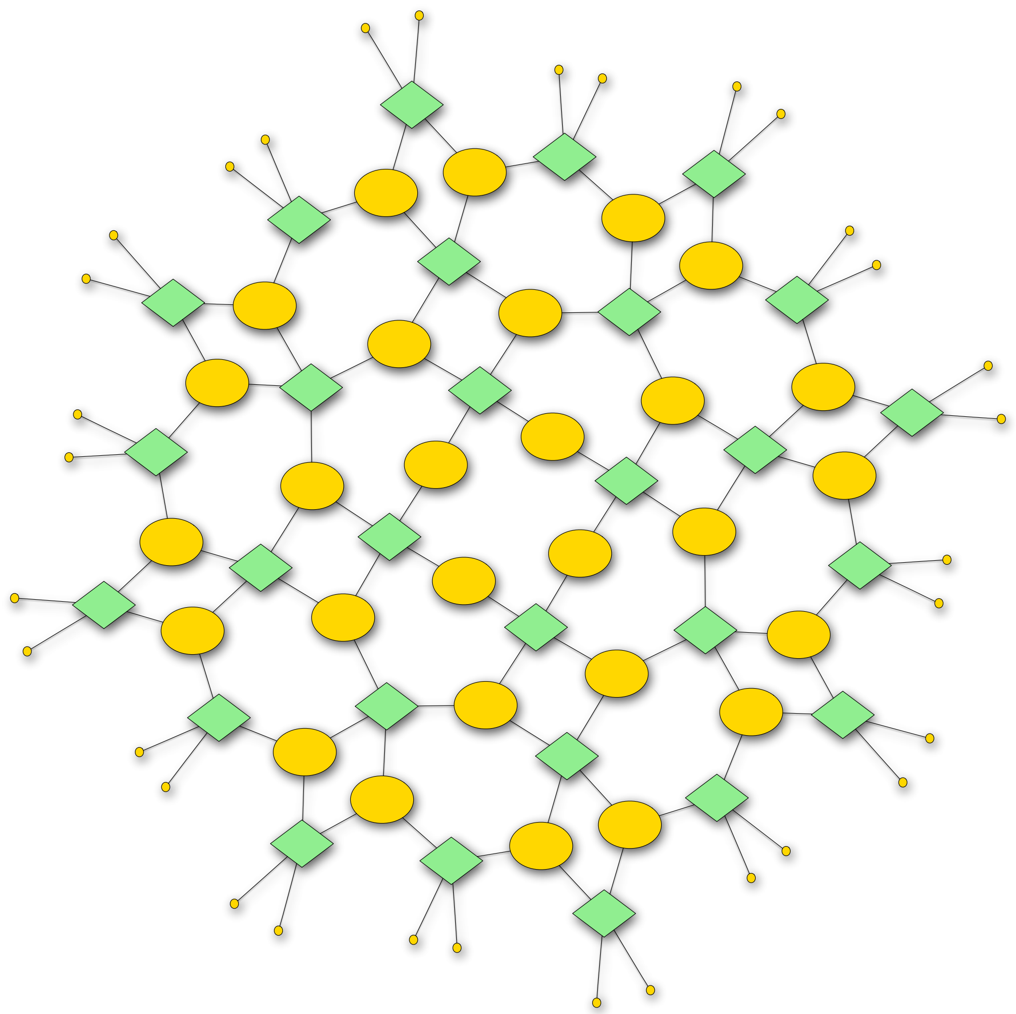


MPS

MERA



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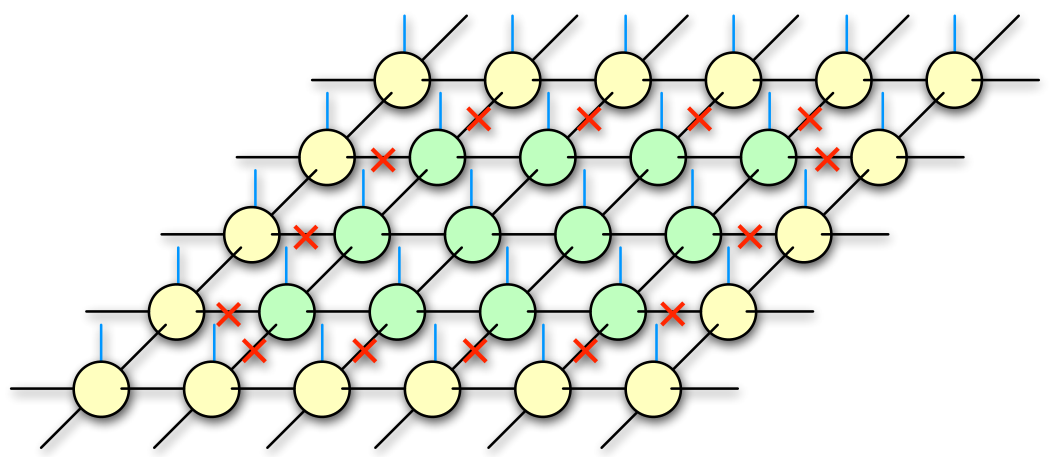

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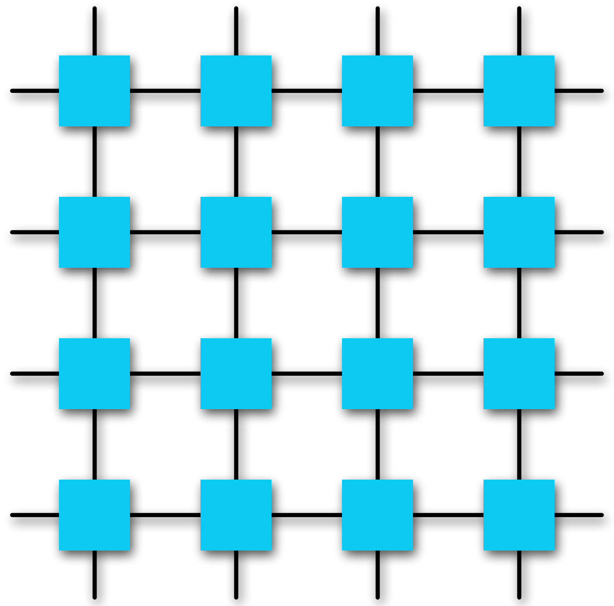

Tensor network, tensor network algorithm, and entanglement flow

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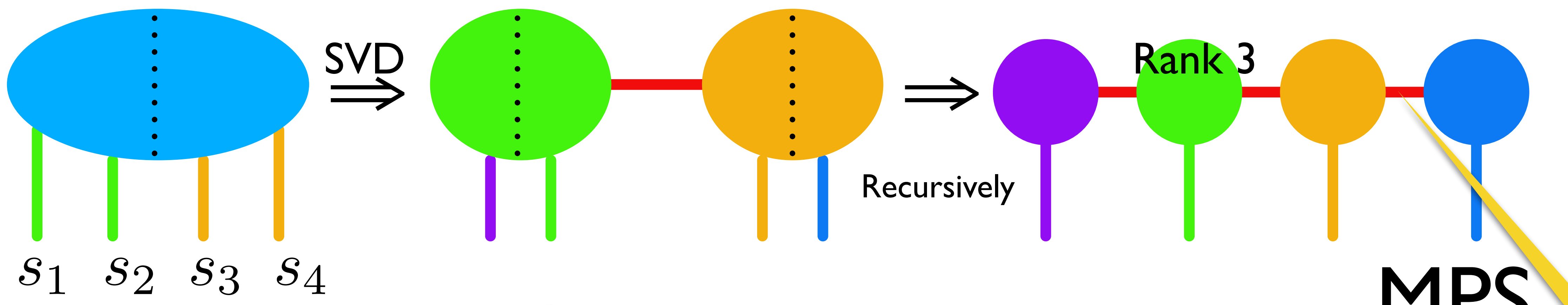

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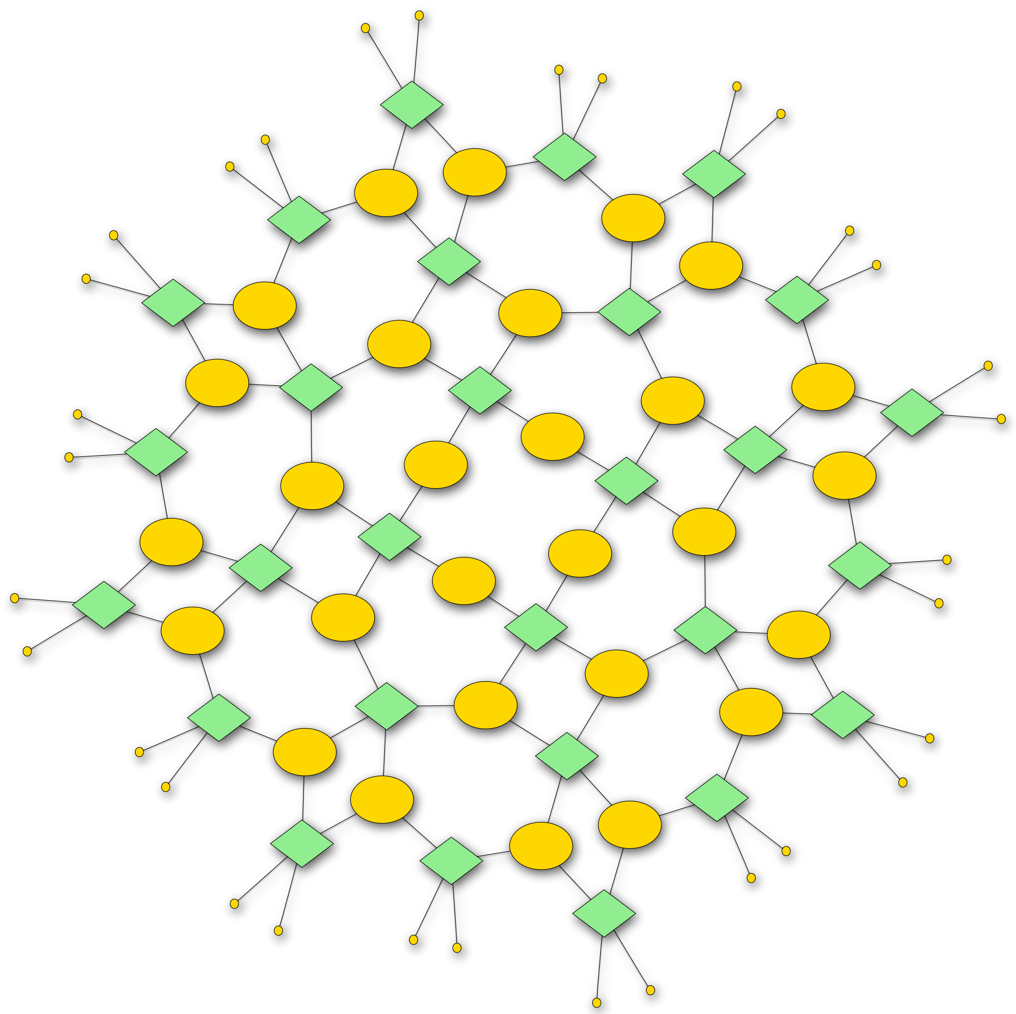

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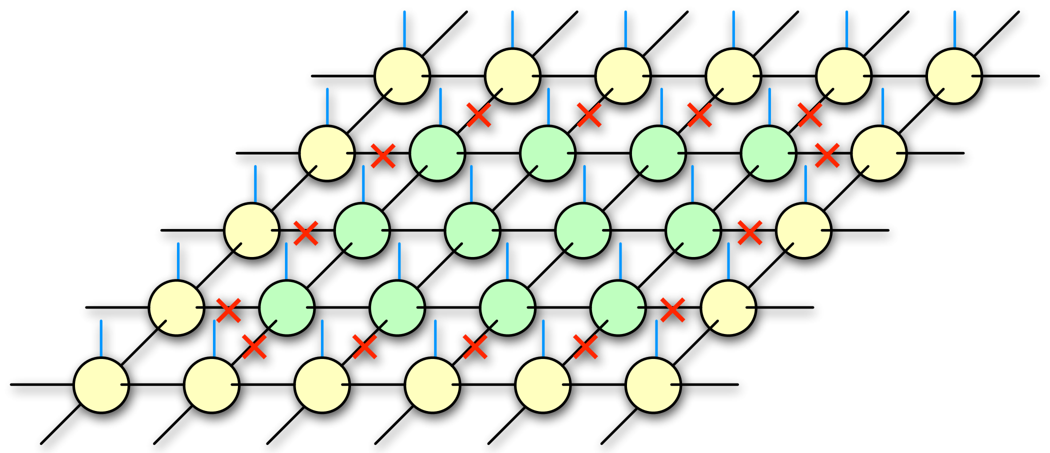

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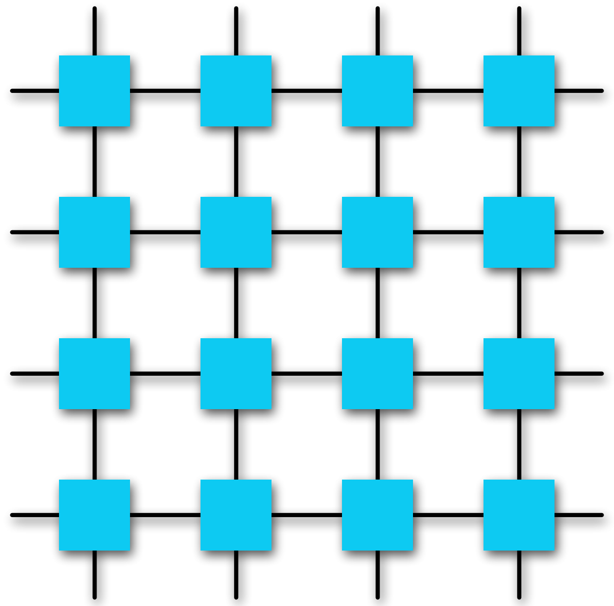

Entanglements flow in a link

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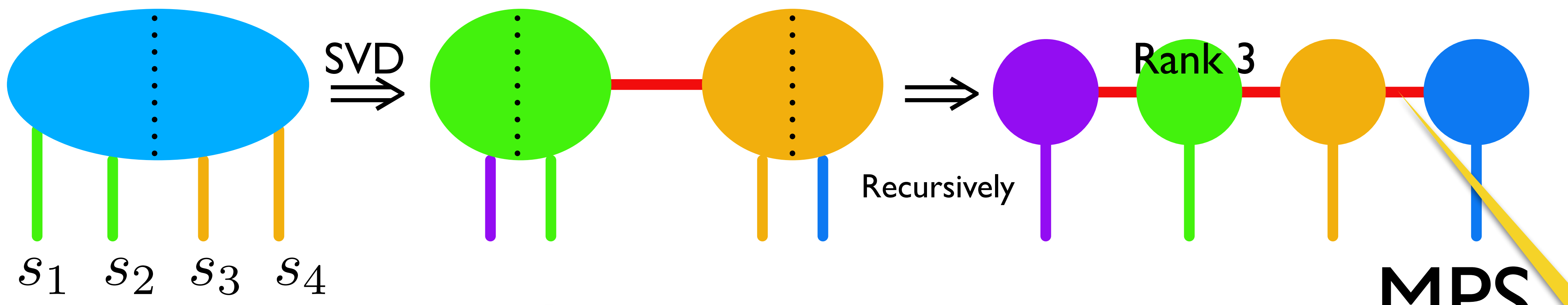

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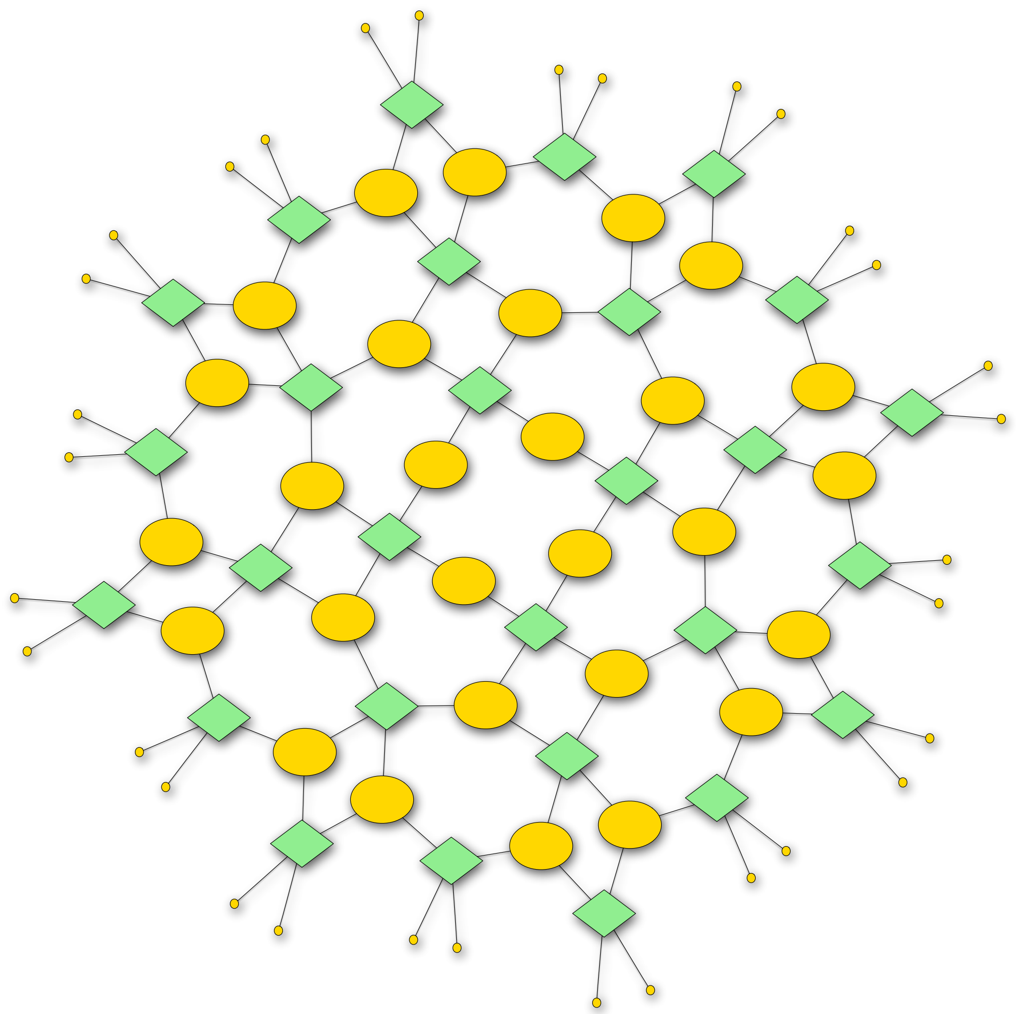

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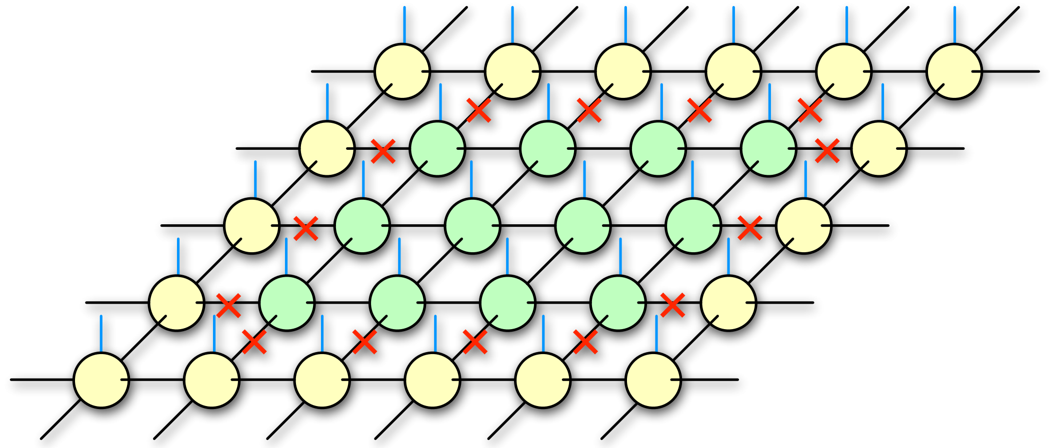

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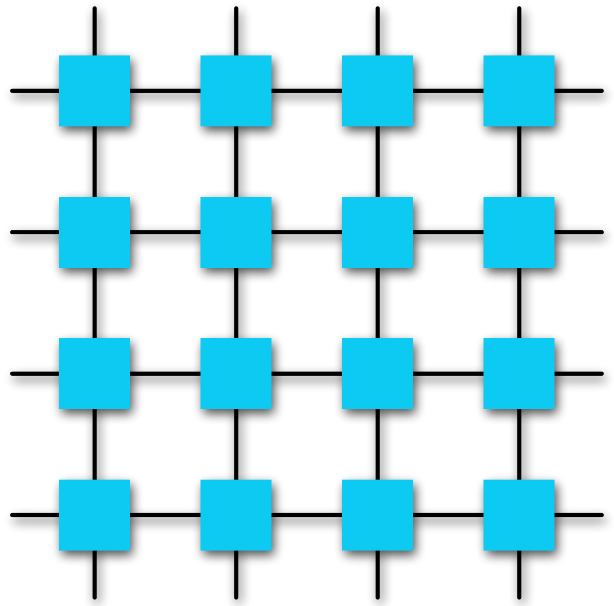


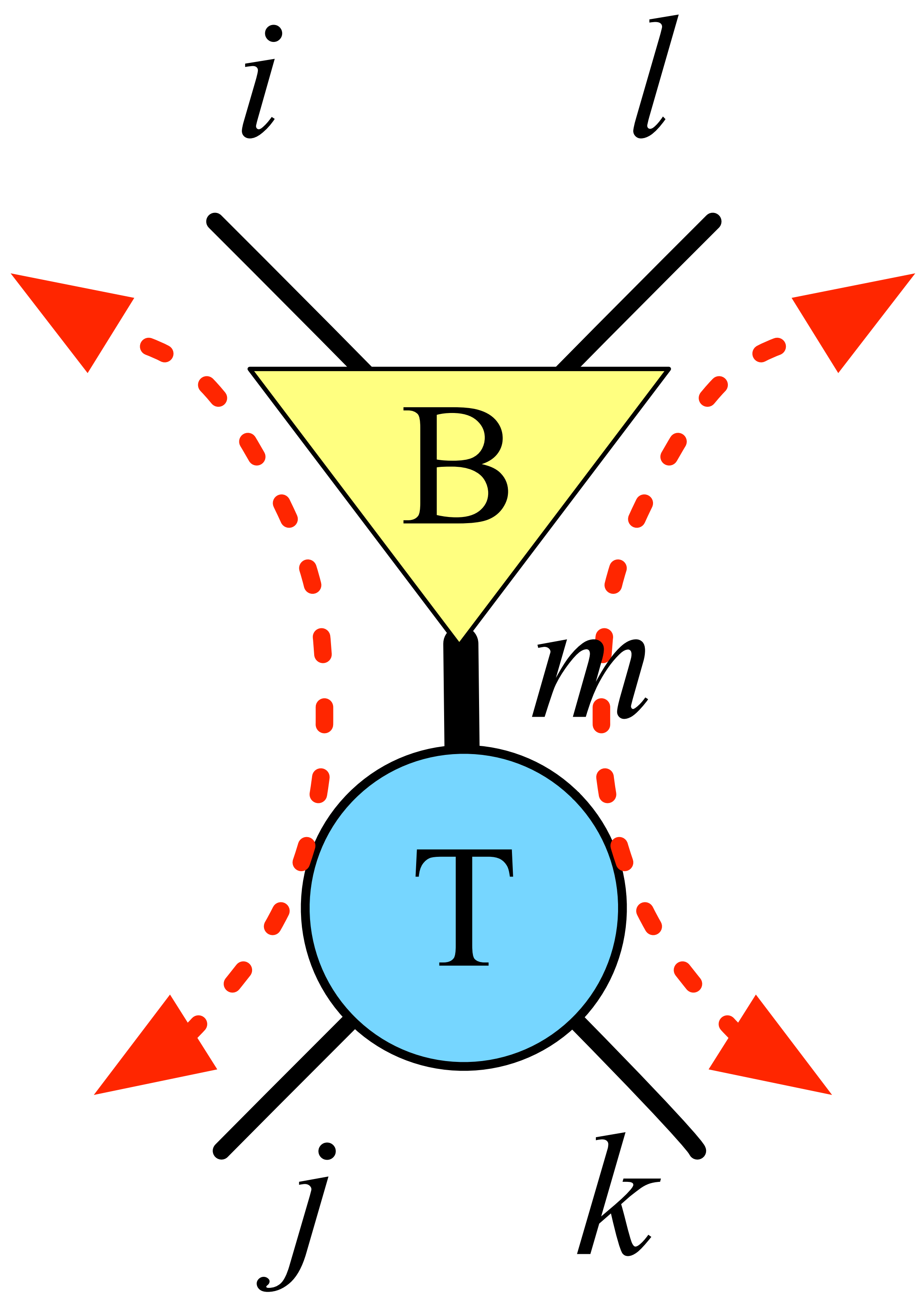
Entanglements flow in a link

Tensor network algorithm

TEBD, CTM, TRG, HOTRG, …

Entanglement branching operator

Split of a composite entanglement flow in a link



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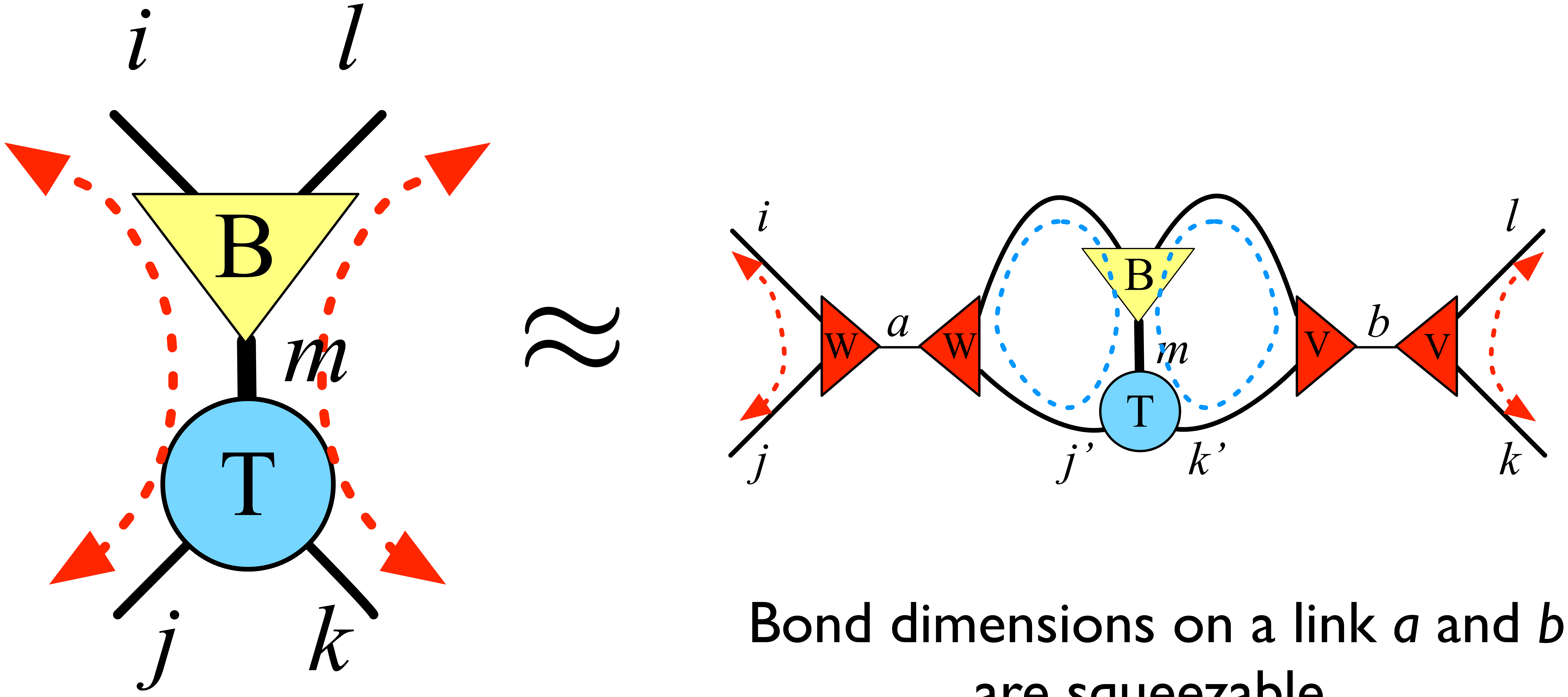
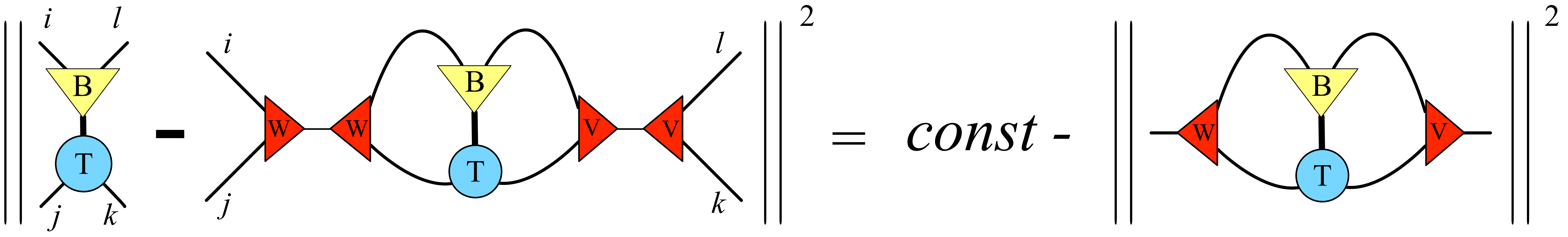


Bond dimensions on a link *a* and *b* are squeezable, when *B*, *W*, and *V* are optimized

Iteration method to solve an optimization problem for entanglement branching operator

Minimize a distance between the following TNs



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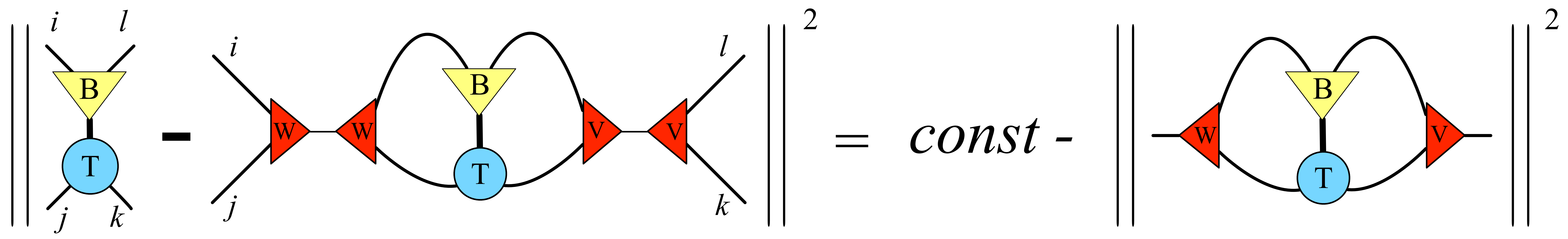


- **Algorithm**

(1) Initialize B, w, and v randomly. Set the values of bond dimension of links a and b one.

(2) Iteratively update B, w, and v to minimize the squared distance.

(3) Increase bond dimensions of links a and b, and extend bond dimensions of w and v.

(4) Go back to (2), until bond dimensions of links a and b reach a limit of them.

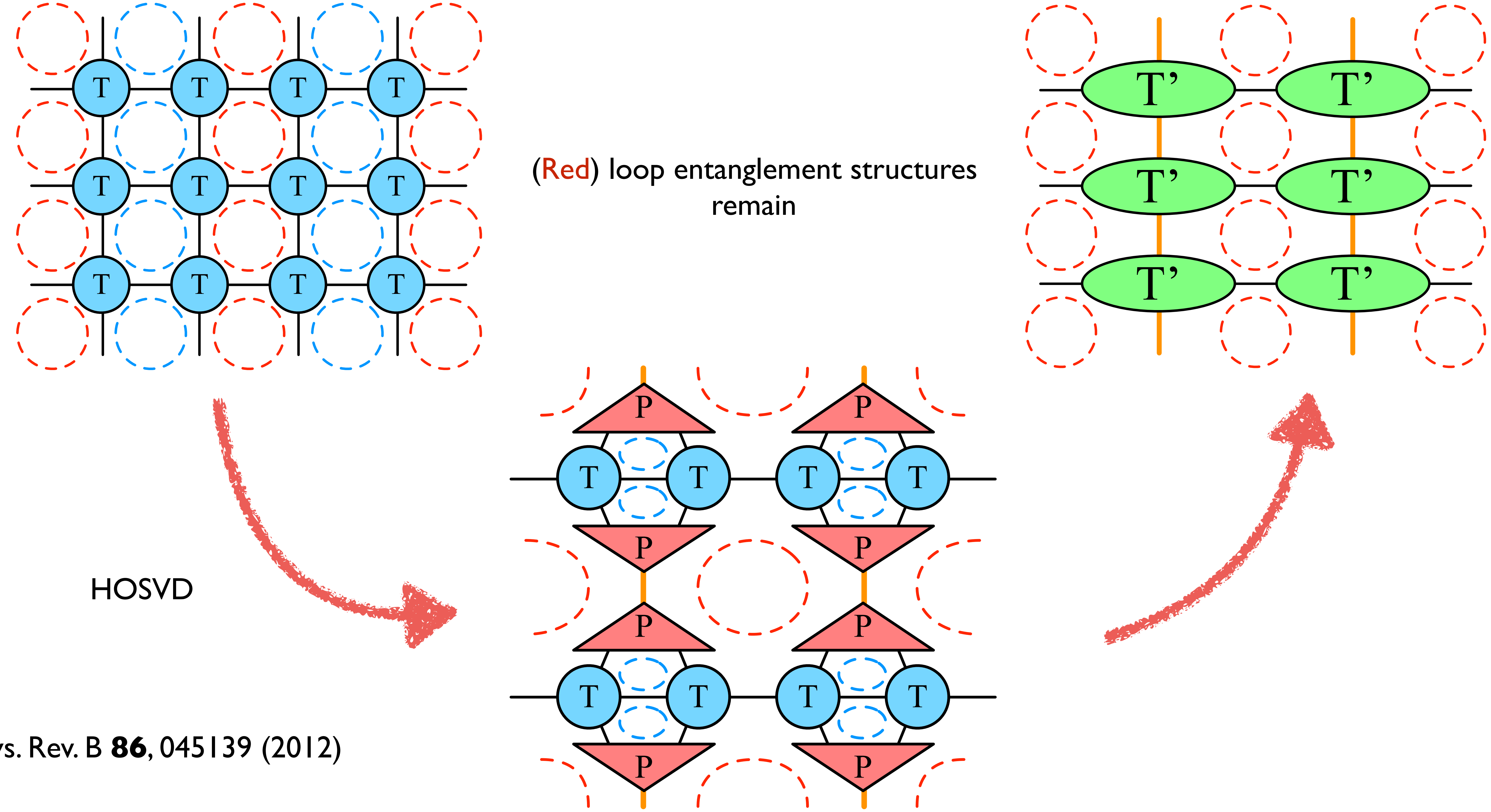
Improvement of HOTRG by entanglement branching

- **Necessary condition of a proper real-space RG**

Gu and Wen, Phys. Rev. B **80**, 155131 (2009)
Evenly and Vidal, Phys. Rev. Lett. **115**, 180405 (2015)

  - erase entanglements under a renormalized scale ⟶ TNR based on TRG (not HOTRG)

- **HOTRG algorithm**



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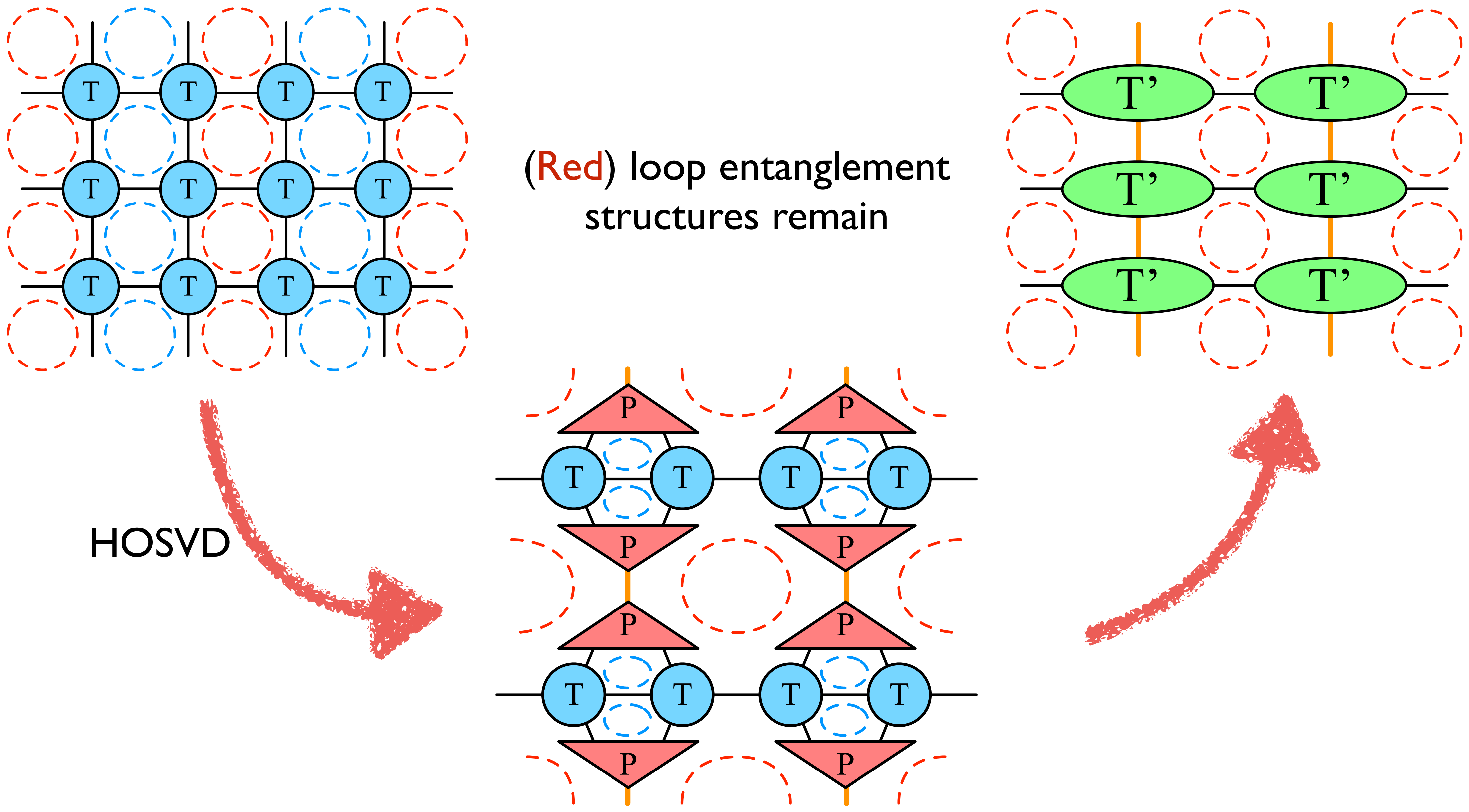

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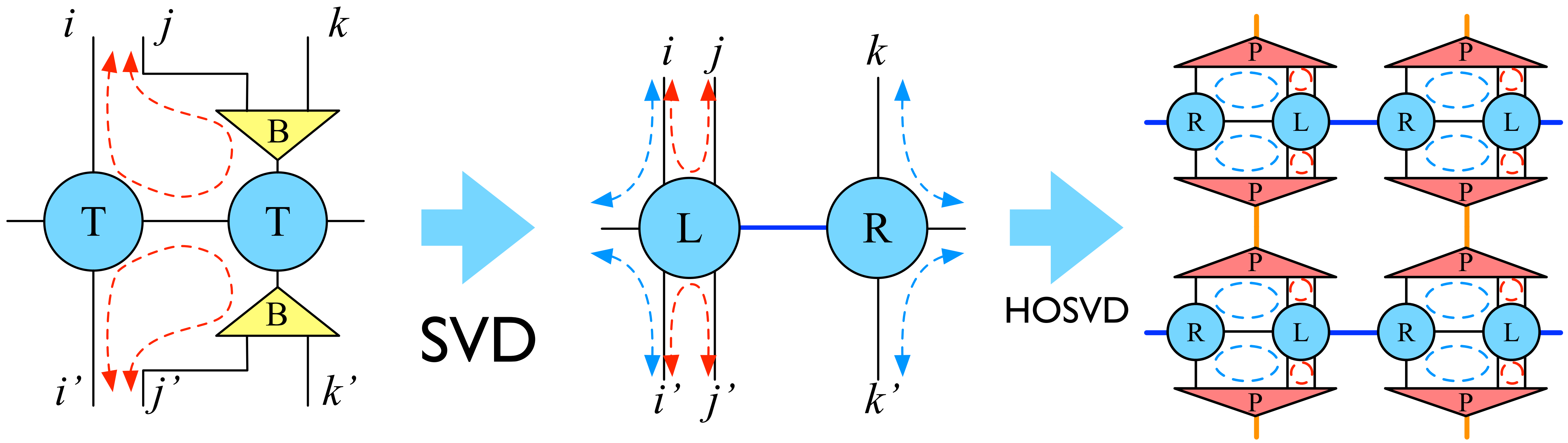
- **Pick up a red entanglement flow**



There is no entanglement between *L* and *R*.

Gather loop entanglement structures in the combination of *R* and *L*.

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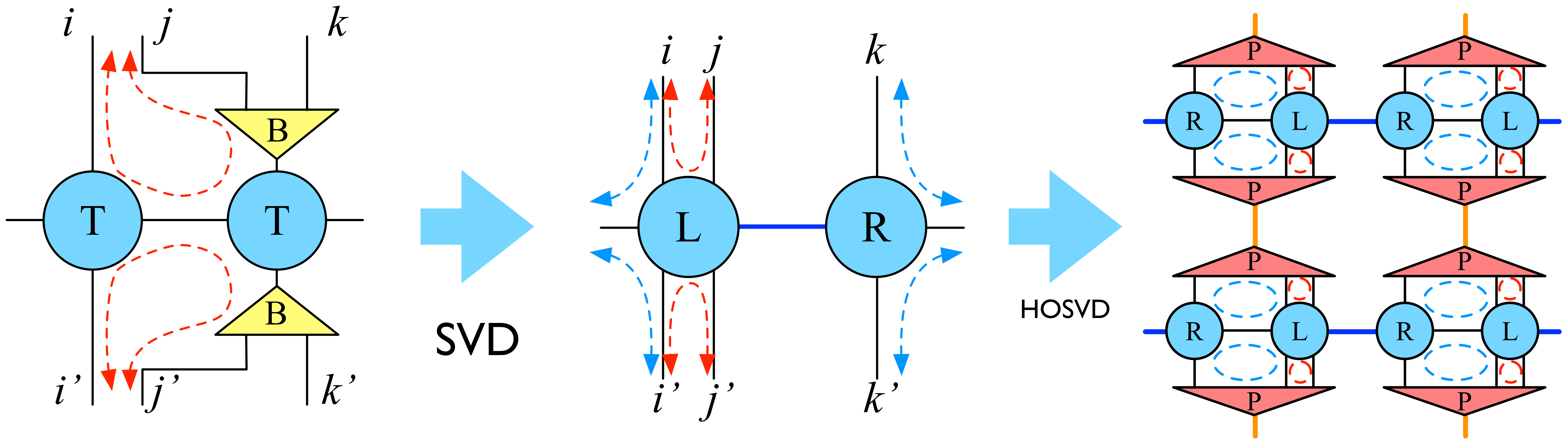

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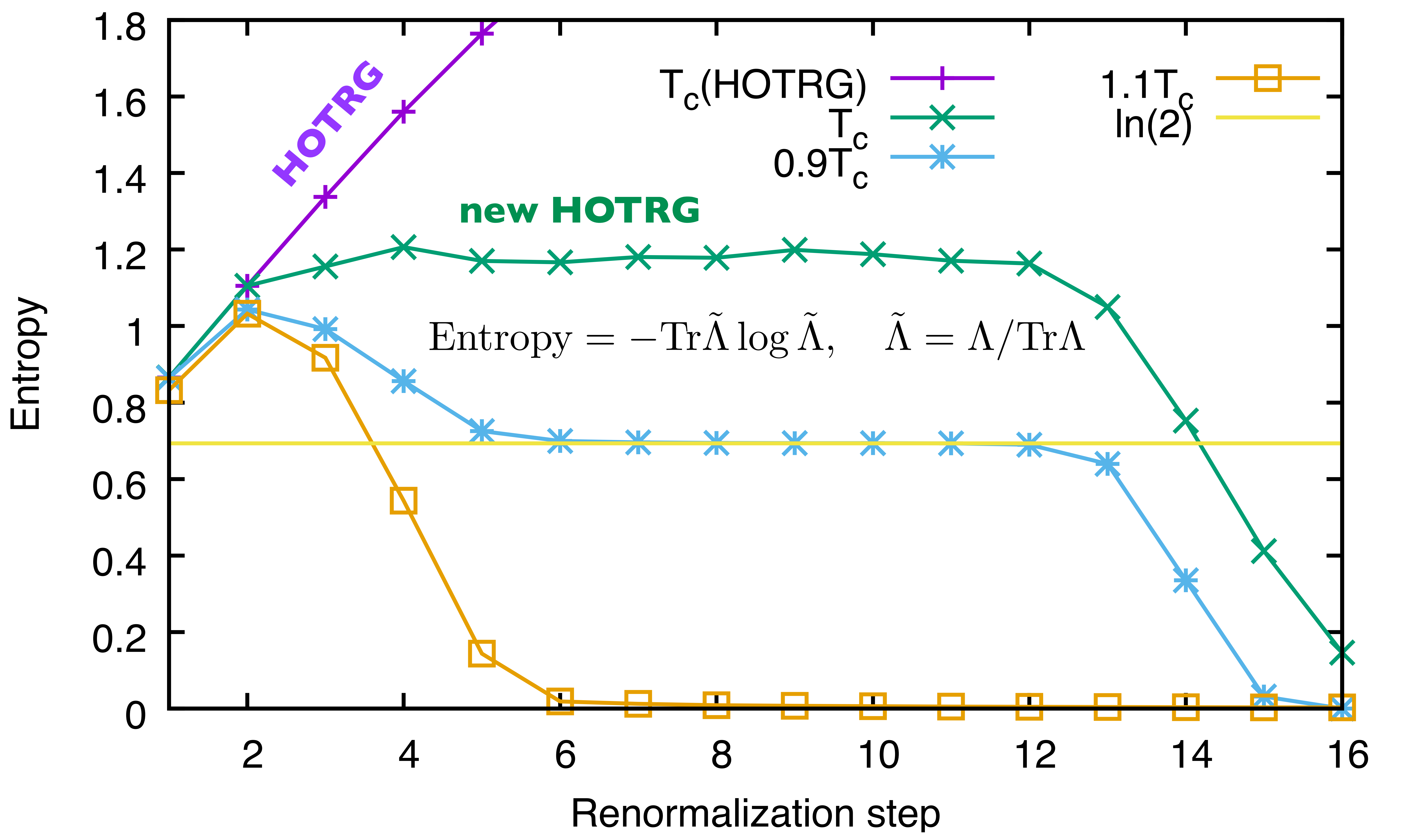
Example: HOTRG of 2D Ising model



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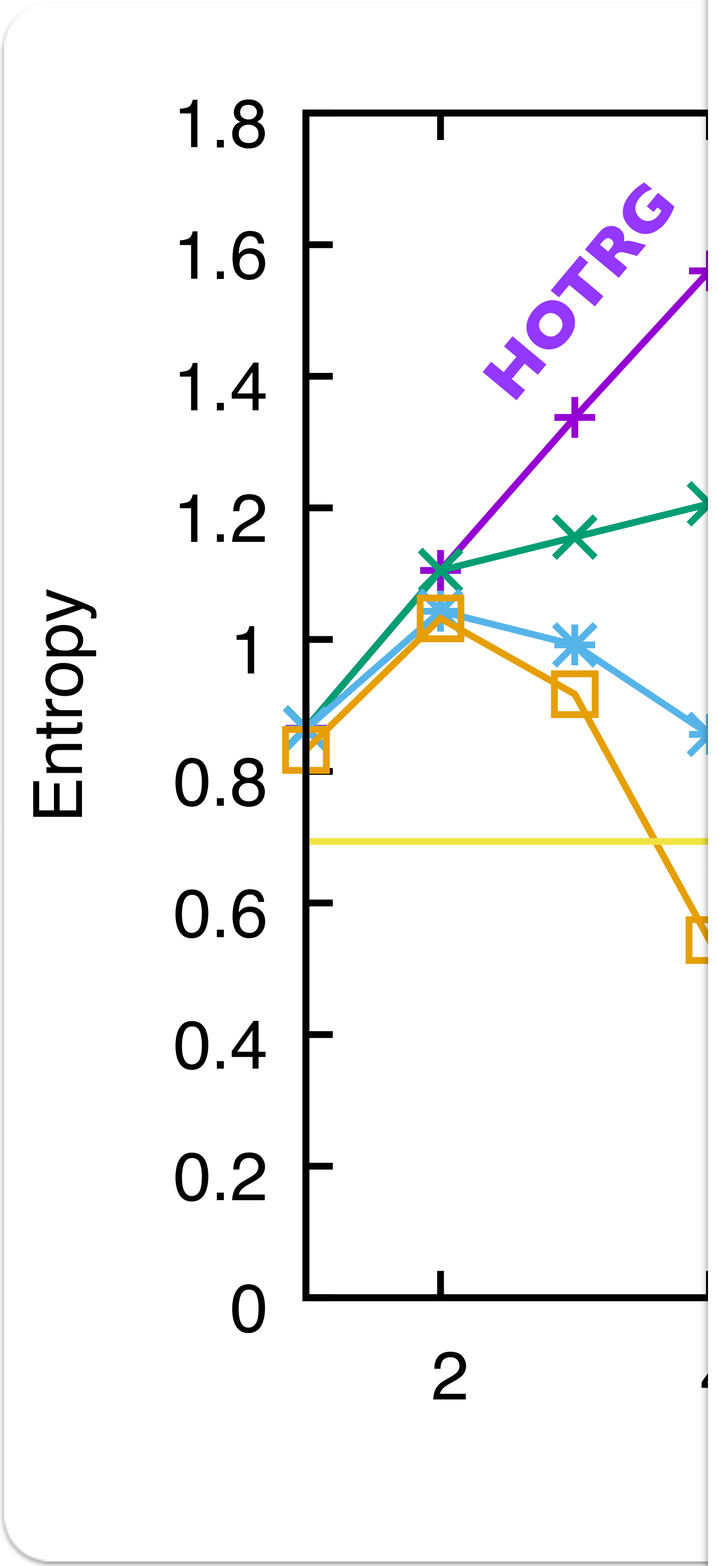

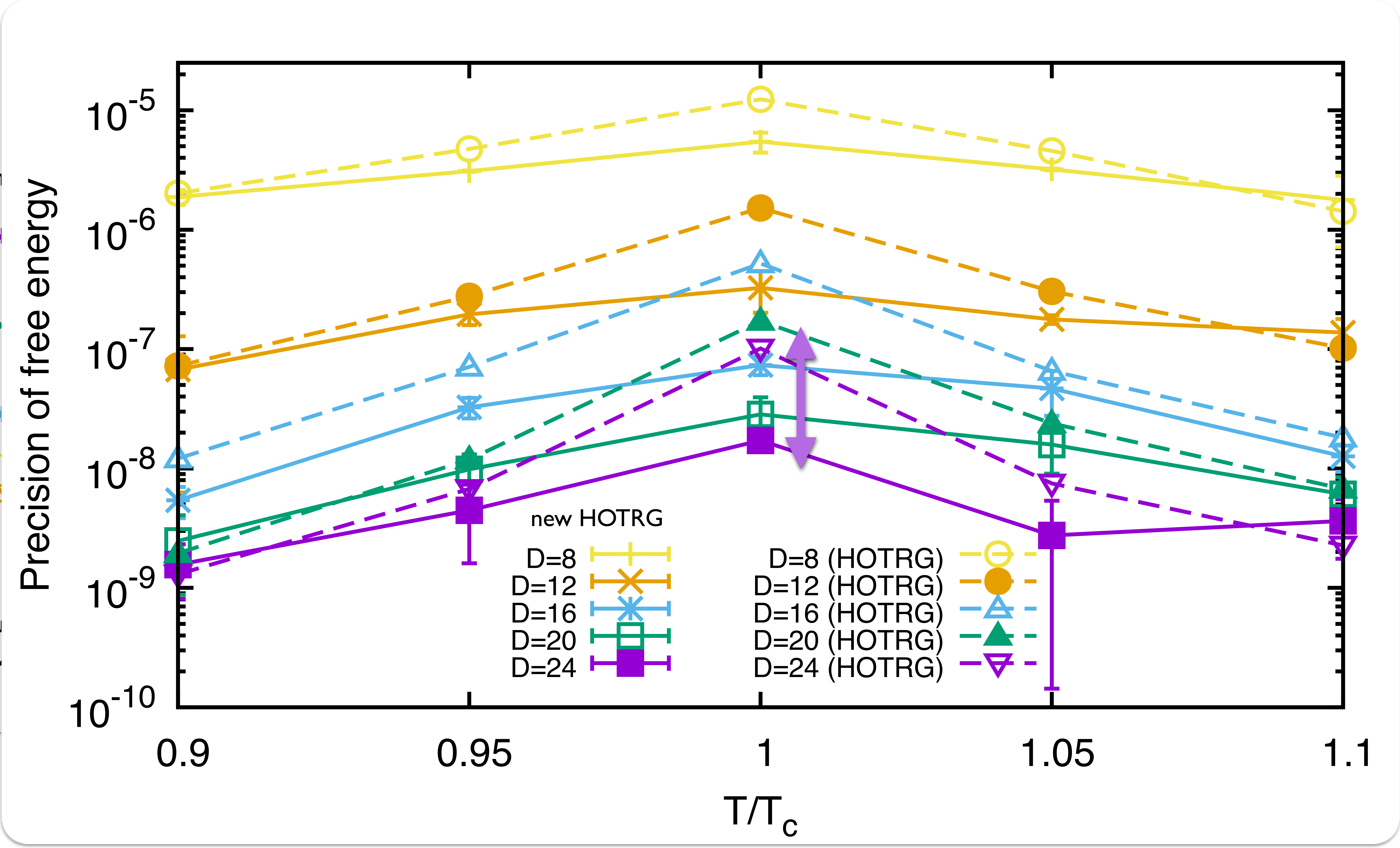

Many-body decomposition

Many-body decomposition

- **Tensor decomposition**

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  - Matrix-based decomposition yields only a two-body tensor network

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  - Many-body decomposition by entanglement branching operator

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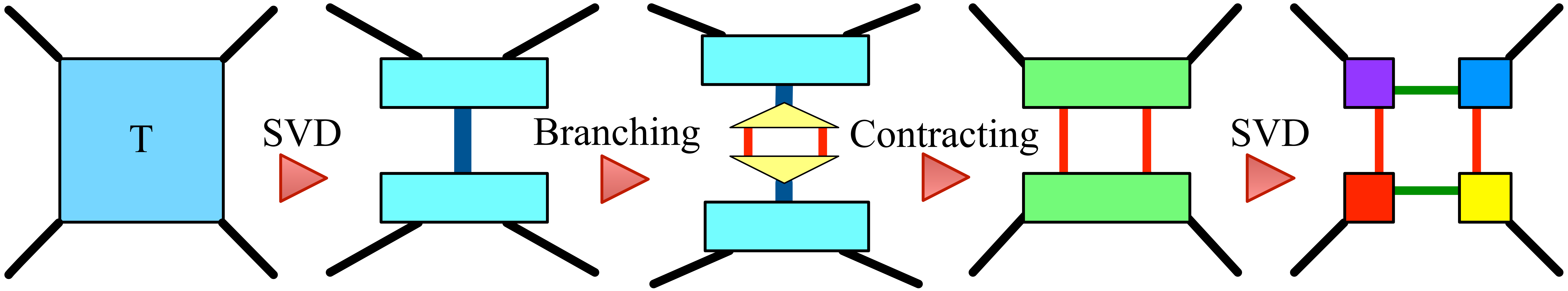
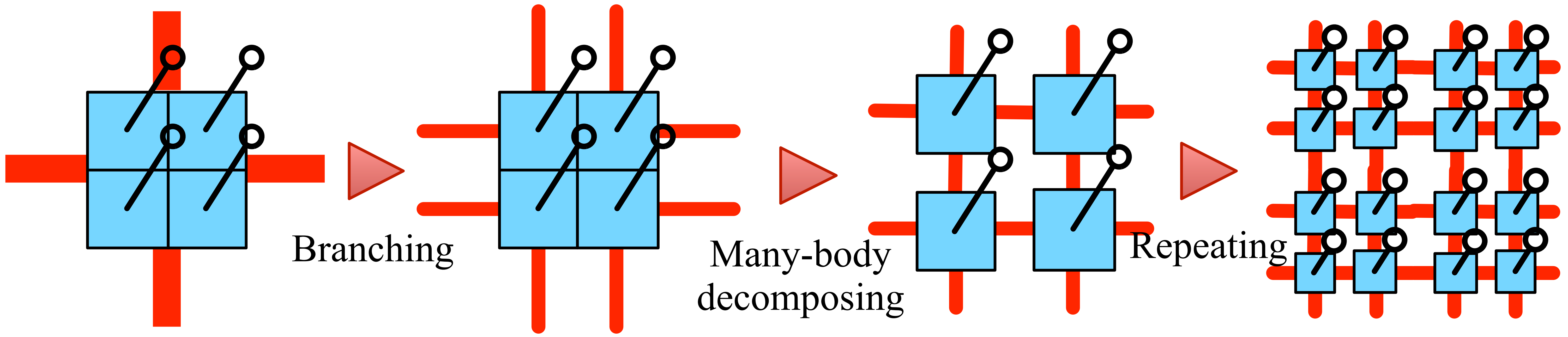
Many-body decomposition and derivation of PEPS

- **Tensor decomposition**
  - Matrix-based decomposition yields only a two-body tensor network
  - Many-body decomposition by entanglement branching



- **Derivation of PEPS based on many-body decomposition**

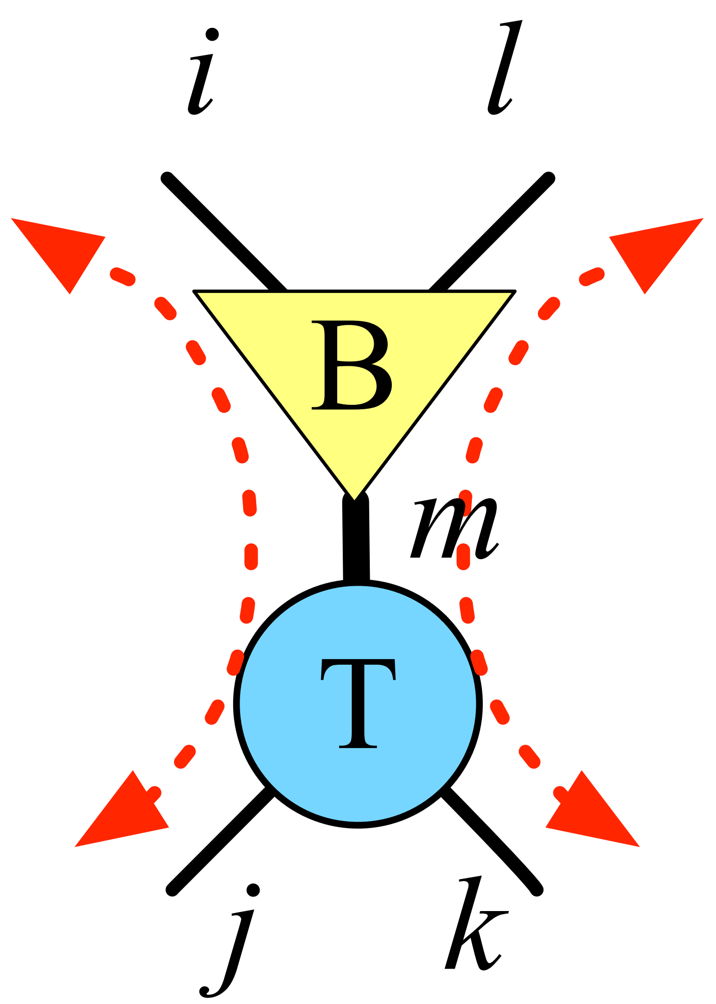


If the area law of entanglement entropy holds, bond dimensions of a derived PEPS are finite

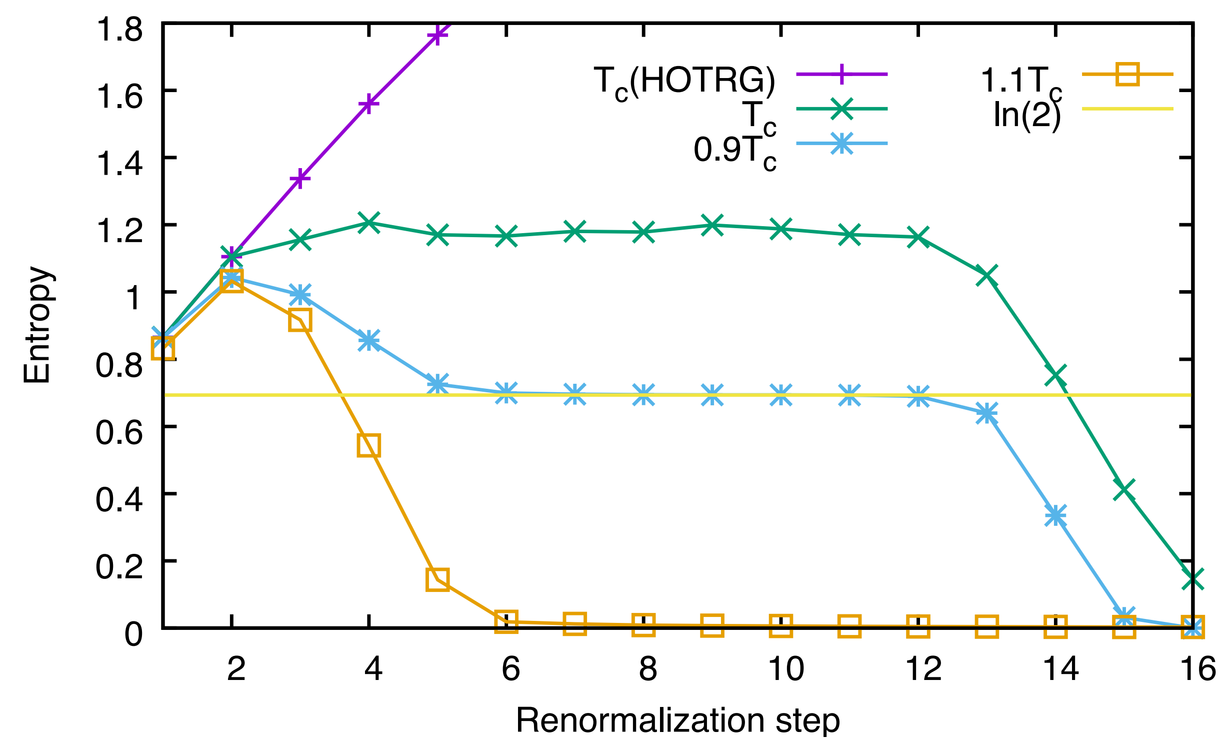
The metric in PEPS is related to entanglement strength

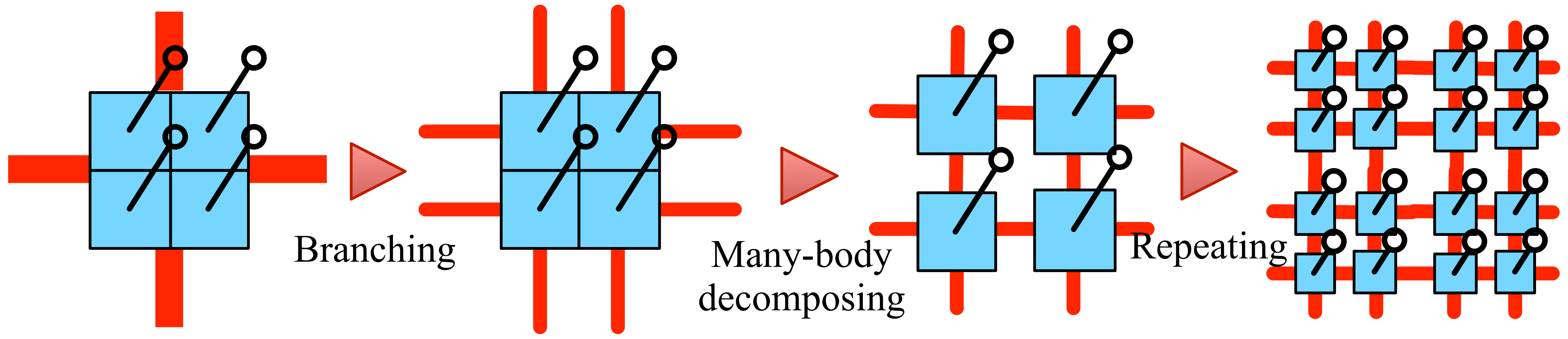
Summary

- **Entanglement branching operator**
  - split of a composite entanglement flow in a link
  - optimization problem by squeezing operators for EB operator
    - iteration method can be applied



- **Applications of entanglement branching operators**
  - improvement of HOTRG
    - proper RG
    - new tensor network state
  - many-body decomposition
    - derivation of PEPS

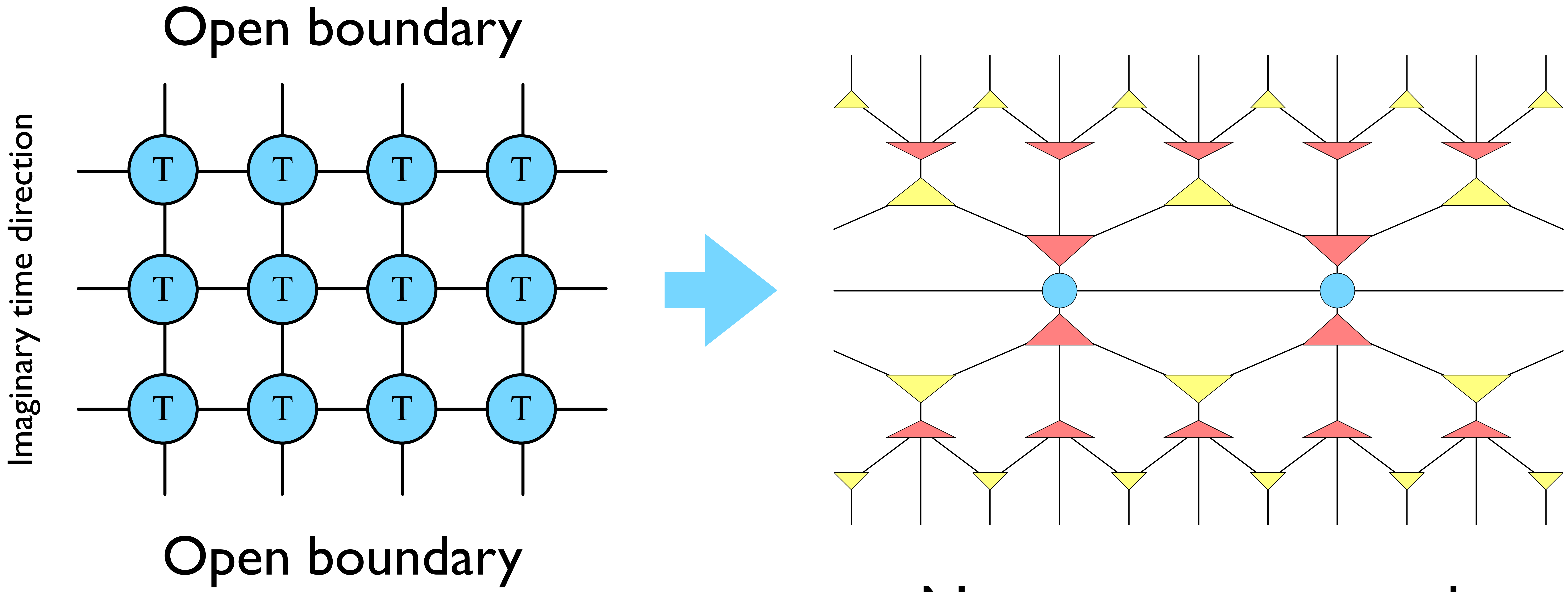




Reference: Kenji Harada, "Entanglement branching operator", Phys. Rev. B **97**, 045124 (2018)

New tensor network state as like MERA

- **Repeating a new HOTRG procedure to a tensor network representation of a density operator**



New tensor network

Log correction of E.E. : ok!